Questions and directions for future work

Lenny Ng

Knot contact homology is an invariant of knotted submanifolds in R^n obtained by counting holomorphic curves in the cotangent bundle of R^n with certain boundary conditions. For knots in R^3, this produces a fairly strong knot invariant that can be formulated combinatorially. In this talk, I describe how this invariant is related to the fundamental group of the knot complement via "string topology", and deduce an elementary proof that knot contact homology detects the unknot. This is partly joint work with Kai Cieliebak, Tobias Ekholm, and Janko Latschev.

There are a number of interesting open questions in this subject. To me, the most accessible is the following: is knot contact homology a complete invariant, that is, is it always different for nonisotopic knots? One consequence of the string-topology approach is a formulation of degree-0 knot contact homology as a certain subring of the group ring of the knot group. It seems conceivable that one could use the fact that the knot group and peripheral subgroup are a complete knot invariant to deduce the same for knot contact homology. This would be a striking validation of Arnold's philosophy that the symplectic topology of cotangent bundles encodes the smooth topology of the underlying manifold.

Another direction for future study is to extend the string-topology approach to more general settings, e.g., knotted submanifolds in arbitrary-dimensional manifolds. This might allow us to use contact topology to distinguish exotically embedded submanifolds such as the Haefliger spheres, or even to approach the smooth four-dimensional Poincare conjecture.