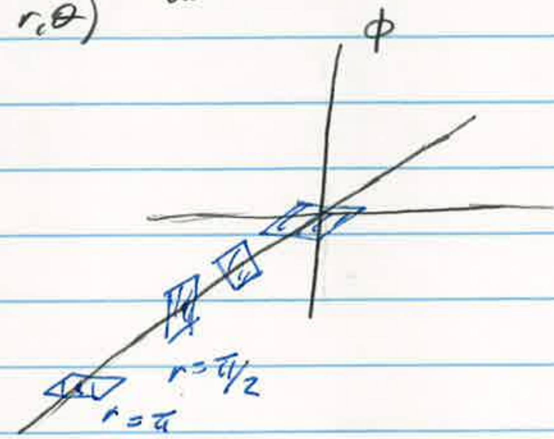
James Conway
TTC 2014

Transverse Surgery on Contact 3-manifolds

Setup (M^3, ξ) • M closed oriented 3-mfd.
• $\xi = \ker\alpha$ $\alpha \in \Omega^1(M)$
s.t. $\alpha \wedge d\alpha > 0$.

eg: $(S^1 \times \mathbb{R}^2, \xi_{tw} = \ker(\cos r\, d\phi + r \sin r\, d\theta))$
(ϕ, r, θ)



Dichotomy: Tight vs. Overtwisted

subtle

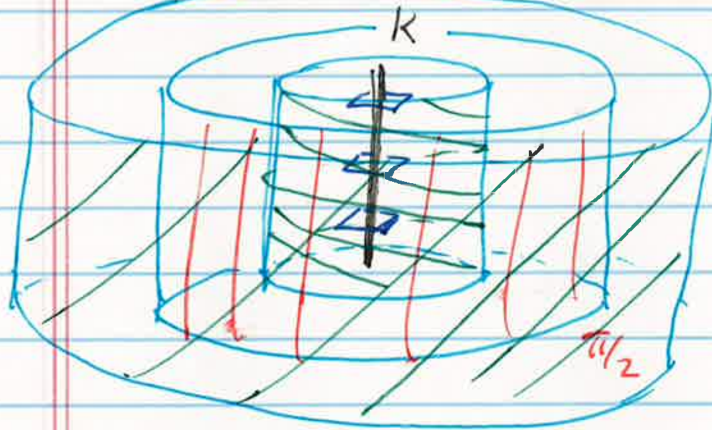
classified via homotopy classes of plane field (Eliashberg '89)

Question: When do operations on tight contact manifolds preserve tightness?

Knots: $K \subseteq (M, \xi)$ is
→ transverse if $\alpha(T_x K) > 0$ ie $K \pitchfork \xi$
→ Legendrian if $\alpha(T_x K) = 0$. i.e. $TK \subseteq \xi$.

Given a Legendrian L, we can form a <u>transverse push off</u> $T(L)$, top. isotopic to L

<u>Fact:</u> every transverse knot has a nbhd contactomorphic to $\{r \le a\}$ in $(S^1 \times \mathbb{R}^2, \xi_{rot})$ some a.



"slopes" of $\xi \cap \partial\{r \le a\}$ are decreasing

slope convention

$p\mu + q\lambda \longleftrightarrow p/q$

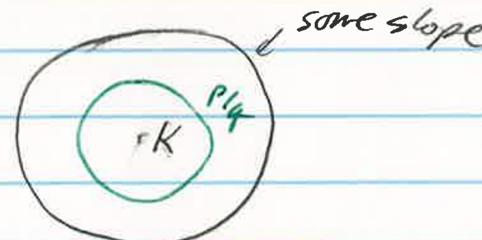
λ = Seifert slope.

<u>Transverse Surgery</u>

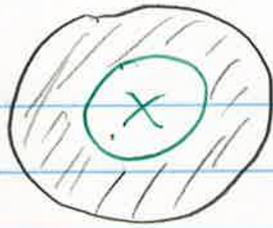
(Martinet, Lutz, Gay, Colin, Baldwin-Etnyre)

<u>admissible surgery</u>

nbhd of K



remove nbhd with ∂ slope p/q

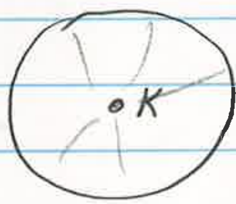


collapse leaves $\longrightarrow$ $(M_{p/q}(K), \xi_{p/q}(K))$

contact cut (Lerman)

$\Rightarrow$ depends on nbhd. $\Leftarrow$

Indmissible Surgery:



nbhd $\longrightarrow$ remove K

$\longrightarrow$ add in $T^2 \times I$ layer

$\longrightarrow$ collapse leaves $(M_{p/q}(K), \xi^{i}_{p/q}(K))$

Surgery on Legendrian knots:

given a Legendrian get 2 invts

tb $\leftarrow$ Thurston-Bennequin $\leftarrow$ "contact framing"

r $\leftarrow$ rotation number $\leftarrow$ relative Euler class

surgery on L: pos $\geq tb \xleftrightarrow{(c.)}$ inadmissible surgery on $T(L)$
(with $-$ sign stabilization)

$\leq tb \longrightarrow$ admissible
neg

(Gay, Baldwin–Etnyre)

$\not\longleftrightarrow$

<u>Question:</u> $\pitchfork$ surgery <u>preserve tightness</u>?

neg. Leg surgery $\rightsquigarrow$ always (Wand)
admissible $\pitchfork$ surgery $\rightsquigarrow$ not always (Baldwin – Etnyre)

inadmissible $\pitchfork$ $\rightarrow$ in general <u>no</u>
sometimes yes

<u>Two Tools:</u>

1) open book decomositions

$K \subseteq M$ fibered
binding

$\rightsquigarrow \Sigma \rightarrow M \setminus K$
page. $\downarrow$
S^1

$\rightsquigarrow$ contact str. ξ_K on M
supported by open book

and K is a $\pitchfork$ knot in ξ_K

2) Heegaard Floer: (Ozsváth-Szabó)

$$M^3 \rightsquigarrow \widehat{HF}(M) \text{ vector space over } \mathbb{Z}_2$$

$$(M,\xi) \rightsquigarrow c(\xi) \in \widehat{HF}(-M)$$

$$c(\xi) \neq 0 \Rightarrow \xi \text{ tight}$$

Past Results:

Golla: exactly when integral inadmissible π surgery on knot in (S^3, ξ_{std}) has $c(\xi_n^i) \neq 0$.

Lisca-Stipsicz: If $L \subseteq (S^3, \xi_{std})$ Legendrian and $tb \leq -2$ then $c(\xi^i_{(tb+1)}(T(L))) = 0$

Hedden-Plamenevskaya:

If $K \subseteq M$ fibered, $c(\xi_K) \neq 0$
then $c((\xi_K)^i_r(K)) \neq 0 \ \forall r \geq 2g(K)$
rational

Th$^{\underline{m}}$ (C):

$L \subseteq (M,\xi)$ Legendrian, null-homologous $tb(L) \leq -2$ and $|rot(L)| \geq 2g(L) + tb(L)$ then $\xi^i_{tb+1}(L)$ is overtwisted

<u>Thm (C.)</u>: $K \subseteq M$ fibered, ξ_K tight
then $(\xi_K)_r^{?'}(K)$ is tight $\forall r > 2g(K)-1$