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Auscher, Pascal (F-PCRD); **Hofmann, Steve** (1-MO); **Lacey, Michael** (1-GAIT); **McIntosh, Alan** (5-ANU); **Tchamitchian, Ph.** (F-XMAR3)

The solution of the Kato square root problem for second order elliptic operators on $\mathbb{R}^n$. (English summary)

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FEATURED REVIEW.

The papers under review provide the complete solution to well-known problems which originate in the work of T. Kato [J. Math. Soc. Japan **13** (1961), 246–274; MR **25** #1453] on fractional powers of dissipative operators. The questions were originally posed by Kato for the general class of abstract maximal accretive operators. A counterexample to the abstract problem was found by J.-L. Lions [J. Math. Soc. Japan **14** (1962), 233–241; MR **27** #2850], and, for the case of maximal accretive operators arising from a form, by A. McIntosh [Proc. Amer. Math. Soc. **32** (1972), 430–434; MR **44** #7354]. However, as McIntosh pointed out [in *Miniconference on operator theory and partial differential equations (Canberra, 1983)*, 124–136, Austral. Nat. Univ., Canberra, 1984; MR 85h:47016], in posing this problem, Kato had been motivated by the special case of elliptic operators, and by the applicability of a positive result, in that special case, to the perturbation theory for parabolic and hyperbolic evolution equations [see J. Math. Soc. Japan **5** (1953), 208–234; MR **15**, 437b]. Thus, McIntosh's [op. cit., 1984] formulation of the "Kato square root problem" was the following:

Let $A = A(x)$ be an $n \times n$ matrix of complex, L^∞ coefficients, defined on $\mathbb{R}^n$, and satisfying the ellipticity (or accretivity) condition (1) $\lambda|\gamma|^2 \leq \mathrm{Re}\langle A\gamma, \gamma\rangle$ and $|\langle A\gamma, \gamma\rangle| \leq \Lambda|\gamma|$, for $\gamma \in \mathbb{C}^n$, where $0 < \lambda \leq \Lambda < \infty$ are constants and $\langle\, ,\rangle$ denotes the standard inner product in $\mathbb{C}^n$. One then defines a second order, divergence-form operator $Lu \equiv -\mathrm{div}(A(x)\nabla u)$, which is interpreted in the usual weak sense via a sesquilinear form.

The accretivity condition (1) enables one to define a square root $L^{\frac{1}{2}}$, and the "Kato square root problem" becomes: $(\mathrm{K})_0$ Show that there exists $C = C(n, \lambda, \Lambda)$ such that, for any such L, we have the estimate $\|L^{\frac{1}{2}}f\|_{L^2(\mathbb{R}^n)} \leq C\|\nabla f\|_{L^2(\mathbb{R}^n)}$. Because the hypotheses of $(K)_0$ are also satisfied by L^*, the adjoint to L, an immediate consequence of $(K)_0$ is that, in fact, $\|L^{\frac{1}{2}}f\|_{L^2(\mathbb{R}^n)} \simeq \|\nabla f\|_{L^2(\mathbb{R}^n)}$, and hence, the domain of the square root of L is the Sobolev space $H^1(\mathbb{R}^n)$.

Note that $(K)_0$ always holds when A is self-adjoint. In his applications to the perturbation theory for hyperbolic equations [op. cit., 1953], Kato was interested in the analyticity of the mapping $A \mapsto L^{\frac{1}{2}}$, at a self-adjoint A. This leads to: $(K)_1$ Let $A_z,\ z \in \mathbb{C}$, denote a family of accretive matrices (i.e. verifying (1) above), which in addition are holomorphic in z, and self-adjoint for real z. Let $L_z = -\mathrm{div}\, A_z\nabla$. Is $L_z^{\frac{1}{2}}$ holomorphic in z, in a neighborhood of $z = 0$? This question is easily seen to be equivalent to the validity of $(K)_0$ for $A = A_0 + \varepsilon B$, where $\|B\|_{L^\infty} \leq M,\ A_0$ is self-adjoint, and ε is small enough.

In the first paper under review, the authors provide a positive answer to $(K)_1$, while in the other two papers the authors provide a positive answer to the more general question $(K)_0$.

These problems have a long history, and a number of people have contributed to their solution. First, R. R. Coifman, McIntosh and Y. Meyer [Ann. of Math. (2) **116** (1982), no. 2, 361–387; MR 84m:42027] proved $(K)_0$ when $n = 1$, simultaneously with their proof of the L^2-boundedness of the Cauchy integral along a Lipschitz curve. In fact, the first argument that the authors of that paper found to prove the boundedness of the Cauchy integral went through a proof of $(K)_0$ when $n = 1$ and an interpolation argument. (This is an unpublished proof.) The equivalence between $(K)_0$ when $n = 1$ and the boundedness of the Cauchy integral on Lipschitz curves was actually proved in [C. E. Kenig and Y. F. Meyer, in *Recent progress in Fourier analysis (El Escorial, 1983)*, 123–143, North-Holland, Amsterdam, 1985; MR 87h:47113; see also P. Auscher, A. G. R. McIntosh and A. R. Nahmod, Indiana Univ. Math. J. **46** (1997), no. 2, 375–403; MR 98k:47033]. The first positive results in higher dimensions used the technique of multilinear expansions, in an extension of the methods used in the $n = 1$ case. Independently, Coifman, D. Deng and Meyer [Ann. Inst. Fourier (Grenoble) **33** (1983), no. 2, x, 123–134; MR 84h:35040] and E. Fabes, D. Jerison and Kenig [Proc. Nat. Acad. Sci. U.S.A. **79** (1982), no. 18, 5746–5750; MR 83k:47035; Amer. J. Math. **107** (1985), no. 6, 1325–1368 (1986); MR 87e:47070] established $(K)_0$ provided $\|I - A\|_\infty \leq \varepsilon(n)$. The methods used in those papers allowed one to replace the identity matrix I by any constant accretive matrix [see E. B. Fabes, D. S. Jerison and C. E. Kenig, Ann. of Math. (2) **119** (1984), no. 1, 121–141; MR 85h:35069]. G. David and J.-L. Journé [Ann. of Math. (2) **120** (1984), no. 2, 371–397; MR 85k:42041] gave a different proof of the multilinear operator bounds using their $T(1)$ theorem. Finally, sharper bounds for the constant $\varepsilon(n)$ (of the order $n^{-\frac{1}{2}}$) were obtained by Journé [Publ. Mat. **35** (1991), no. 1, 299–321; MR 92d:47050]. The multilinear expansion approach also led to non-perturbative results for operators with “smooth coefficients”. For instance, Fabes-Jerison-Kenig (1982, unpublished) established $(K)_0$ for uniformly continuous coefficients, and L. Escauriaza (1994, unpublished) for VMO coefficients, while McIntosh [J. Funct. Anal. **61** (1985), no. 3, 307–327; MR 87e:47068] considered coefficients that are pointwise multipliers of some Sobolev space $H^s, s > 0$. However, the scope of these ideas seemed limited, and a new approach was needed in order to make further progress on these problems. The first breakthrough came in the work of Auscher and Ph. Tchamitchian [Astérisque No. 249 (1998), viii+172 pp.; MR 2000c:47092] (hereafter [AT]), who in analogy with the $T(b)$ theorems for singular integrals [see A. G. R. McIntosh and Y. F. Meyer, C. R. Acad. Sci. Paris Sér. I Math. **301** (1985), no. 8, 395–397; MR 87b:47053; G. David, J.-L. Journé and S. W. Semmes, Rev. Mat. Iberoamericana **1** (1985), no. 4, 1–56; MR 88f:47024; M. Christ, Colloq. Math. **60/61** (1990), no. 2, 601–628; MR 92k:42020; R. R. Coifman and Y. F. Meyer, in *Beijing lectures in harmonic analysis (Beijing, 1984)*, 3–45, Ann. of Math. Stud., 112, Princeton Univ. Press, Princeton, NJ, 1986; MR 88e:42030] introduced a "$T(b)$ theorem for square roots", which reduced matters to a suitable "Carleson measure estimate". In this approach two features were very important: (a) the use of the functional calculus, and in particular (b) the use of Gaussian estimates for the heat kernel associated to L, i.e. if $W(t^2,x,y)$ is the kernel of the operator e^{-t^2L} we say that L satisfies "property (G)" if there exist positive constants α,β such that

$$\text{(G1)}\qquad |W(t^2,x,y)|\le \beta t^{-n}\exp\left\{-\frac{|x-y|^2}{\beta t^2}\right\},$$

$$\text{(G2)}\quad |W_{t^2}(x+h,y)-W_{t^2}(x,y)| + |W_{t^2}(x,y+h)-W_{t^2}(x,y)| \le \beta\frac{|h|^\alpha}{t^{\alpha+n}}\exp\left\{-\frac{|x-y|^2}{\beta t^2}\right\}.$$

The initial impact of this point of view, pursued in [AT], was a unified proof for all earlier results, and some improvements of them (for instance, VMO was improved to a bigger subspace of BMO, called ABMO). Moreover, it was also shown in [AT] that special structure conditions on the matrix A did lead to non-perturbation results, without "smoothness" conditions. Nevertheless, it was far from clear from this how to proceed to the general case. In particular, two key difficulties remained. The first and, in retrospect, more fundamental difficulty was to find the "para-accretive" b's for which the "Carleson measure estimate" could be proved. We will dub this (D1).

The second difficulty, which turned out not to be crucial in the end, was what to do when Gaussian estimates were not available, i.e. when property (G) failed. (Examples of such L were known, for instance given in [P. Auscher, T. Coulhon and P. Tchamitchian, Colloq. Math. **71** (1996), no. 1, 87–95; MR 97e:35019], and in fact, related examples were earlier given in [M. Cwikel, E. B. Fabes and C. E. Kenig, in *Conference on harmonic analysis in honor of Antoni Zygmund, Vol. I, II (Chicago, Ill., 1981)*, 557–576, Wadsworth, Belmont, CA, 1983; MR 86d:35022].) We will dub this (D2).

This was the state of affairs until the appearance of the papers under review.

In the first paper under review, the first decisive further progress is made. In this paper the authors settle the question $(\mathrm{K})_1$ in the affirmative. In this setting (small perturbations of self-adjoint operators) (D2) is not present. In fact, the classical works [J. Nash, Amer. J. Math. **80** (1958), 931–954; MR **20** #6592; J. Moser, Comm. Pure Appl. Math. **17** (1964), 101–134; MR **28** #2357; correction; MR **34** #3121; D. G. Aronson, Ann. Scuola Norm. Sup. Pisa (3) **22** (1968), 607–694; MR **55** #8553] established (G1) and (G2) for self-adjoint operators, while the work of Auscher [J. London Math. Soc. (2) **54** (1996), no. 2, 284–296; MR 97f:35034] extended these bounds to their small perturbations. In this paper, the approach is not through the "$T(b)$ theorem for square roots" of [AT], which was mentioned before, but since we are in "small perturbation" situation, it is more in the spirit of the $T(1)$ theorem of David-Journé [op. cit., 1984]. Following [AT], and the proof (see [R. Coifman and Y. Meyer, op. cit., 1986], for instance) of the $T(1)$ theorem, the authors reduce matters to the validity of the following Carleson measure estimate (recall here that $A = A_0 + \varepsilon B,\ A_0$ is self-adjoint, $L = -\operatorname{div} A\nabla,\ L_0 = -\operatorname{div} A_0\nabla$):

$$\|\mu\|_C = \sup_Q \frac{1}{|Q|}\int_Q \int_0^{l(Q)} |te^{-t^2L}\varphi(x)|^2 \frac{dt}{t} dx \le C \tag{2}$$

where Q is a cube in $\mathbb{R}^n$, of side-length $l(Q)$, and $\varphi(x) = x$.

Next, in order to establish (2), they prove the estimate

$$\|\mu\|_C \le C[1+\varepsilon^2]\|\mu\|_C, \tag{3}$$

which for ε small gives the desired result. The key ingredient in establishing (3) is an "extrapolation lemma for Carleson measures" (Lemma 2.5 in the paper), which is combined with the fact that (2) holds for the measure μ_0 corresponding to L_0, and with ideas in the proof of the "$T(b)$ theorem for square roots" of [AT], in order to verify the hypothesis of the "extrapolation lemma". This "extrapolation lemma for Carleson measures" has its origin in the work of J. L. Lewis and M. A. M. Murray [Mem. Amer. Math. Soc. **114** (1995), no. 545, viii+157 pp.; MR 96e:35059] and S. Hofmann and Lewis [Mem. Amer. Math. Soc. **151** (2001), no. 719, viii+113 pp.; MR 2002h:35108] to establish parabolic measure estimates for certain classes of parabolic equations. It utilizes a stopping-time argument, reminiscent of L. Carleson's corona construction [Ann. of Math. (2) **76** (1962), 547–559; MR **25** #5186]. Even though the main results of this paper are in fact superseded by those of the other two papers under review, the reviewer thinks that the "extrapolation lemma" will still prove very useful in the future.

Finally, as mentioned above, a key tool for the verification of the hypothesis of the "extrapolation lemma", inspired by the "$T(b)$ theorem for square roots" [AT], is: if there exists, for each cube Q, a mapping $F_Q\colon 5Q \to \mathbb{C}^n$ (where $5Q$ denotes the concentric dilate of Q having side length $5l(Q)$) such that

$$\int_{5Q} |\nabla F_Q|^2 \le C|Q|, \tag{4}$$

$$\int_{5Q} |L_0 F_Q|^2 \le C\frac{|Q|}{l(Q)^2}, \tag{5}$$

then

$$\frac{1}{|Q|}\int_Q \int_0^{l(Q)} |te^{-t^2L}\varphi(x)P_t(\nabla F_Q)(x)|^2 \frac{dt}{t} dx \le C(1+C+\varepsilon^2\|\mu\|_C) \tag{6}$$

where P_t denotes convolution with a nice approximate identity.

The authors then choose $F_Q = e^{-\eta^2 l(Q)^2 L_0}\varphi(x)$, for η small, and use (6) to verify the hypothesis of the "extrapolation lemma". In some sense, these F_Q play the role of the "b" in the $T(b)$ theorem.

In the second paper under review the authors provide a positive answer to question $(K)_0$, for operators for which the estimates (G1) and (G2) hold, and hence for which (D2) does not arise.

This includes all operators when $n = 2$ [P. Auscher, A. G. R. McIntosh and P. Tchamitchian, J. Funct. Anal. **152** (1998), no. 1, 22–73; MR 99e:47062], and small perturbations of self-adjoint ones (i.e. the case treated in the first paper under review) in all dimensions, by the results of [P. Auscher, op. cit.]. The strategy consists in verifying the hypothesis of the "$T(b)$ theorem for square roots" [AT]. (In fact, it is a small variant of this result that is actually needed.) Thus, in this paper the authors address (D1), with a construction very close in spirit to (4), (5), (6) above. They show that the result follows from the existence of a family $\{F_{\nu,Q}\},\ F_{\nu,Q}\colon 5Q \to \mathbb{C}, \nu \in J$, with J a finite set of cardinality N, such that for each cube $Q \subset \mathbb{R}^n$, one has

$$\int_{5Q} |\nabla F_\nu|^2 \le C|Q|, \tag{7}$$

$$\int_{5Q} |LF_\nu|^2 \le C\frac{|Q|}{l(Q)^2}, \tag{8}$$

$$\sup_Q \int_Q \int_0^{l(Q)} |te^{-t^2L}\varphi(x)|^2 \frac{dt}{t} dx \le \\ C\sum_{\nu\in J}\left\{C_0 + \sup_Q \frac{1}{|Q|}\int_Q \int_0^{l(Q)} |te^{-t^2L}\varphi(x)P_t(\nabla F_{\nu,Q})|^2\frac{dt}{t}dx\right\}. \tag{9}$$

This is the "$T(b)$ theorem for square roots". Then, they construct $F_{\nu,Q}$ in the following way: let $F_Q = e^{-\eta^2 l(Q)^2 L}\varphi(x)$, and for suitable unit vectors ν in $\mathbb{C}^n$, define $F_{\nu,Q} = F_Q \cdot \nu$. Using a stopping time argument, which, in retrospect, originates in the work of Christ [op. cit.], the authors verify the required properties (7), (8), (9). The key point in the proof of the crucial property (9) is that

$$\frac{1}{|Q|}\left|\int_Q (\nabla F - I)dx\right| \le C\eta,$$

where I is the $n \times n$ identity matrix.

In the third paper under review the authors prove that the answer to $(K)_0$ is positive in general. The key new idea in the paper is that, although (G1) and (G2) need not hold in general, there is enough decay, in an averaged sense, to carry out the program developed in the second paper under review. This average decay is similar to estimates first proved by M. P. Gaffney [Comm. Pure Appl. Math. **12** (1959), 1–11; MR **21** #892] and E. B. Davies [J. Anal. Math. **58** (1992), 99–119; MR 94e:58136]. This is implemented by studying average decay and smoothness-decay estimates for the resolvent $(1 + t^2L)^{-1}$ and for $t\nabla(1 + t^2L)^{-1}$. One then uses the resolvent $(1 + t^2L)^{-1}$ in place of the heat operator e^{-t^2L} used before, and manages to carry out the program of the second paper under review. Thus (D2) turns out to be mostly a technical difficulty. Another approach to overcoming (D2), taken in the paper [P. Auscher et al., J. Evol. Equ. **1** (2001), no. 4, 361–385; MR

2003a:35046], is to prove the analogue of $(\mathrm{K})_0$ for elliptic operators of sufficiently high order, for which (G1) and (G2) hold, and then obtaining $(\mathrm{K})_0$ for second-order operators by an interpolation argument of [P. Auscher, A. McIntosh and A. Nahmod, op. cit.].

These three papers provide a complete solution to important questions that arise naturally in partial differential equations, and constitute a beautiful contribution to analysis. They showcase the power of the modern theory of harmonic analysis (such as the $T(1)$, $T(b)$ theorems and the corona decomposition) to prove hard estimates of central importance in partial differential equations.

Reviewed by *Carlos E. Kenig*

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