CS 1050 - Proofs
Homework 10
Assigned Sunday, November 7
Due Thursday**, November 11**

1. Let $f(n) = n$ for all $n \geq 1$ and let $g(n) = n^2$ if $n \geq 1$ and n is even and $g(n) = 1$ if $n \geq 1$ and n is odd.

   Prove that f is *not* $O(g)$. (Warning: you cannot use a limit. Why?)

2. a) Prove that if $f(n) = n!$ and $g(n) = 3^n$, then f is *not* $O(g)$.

   b) Is $g = O(f)$? Prove your answer.

3. Prove the following theorem.

   Theorem 1 *Let* α, β *be any two real numbers such taht* $\alpha \leq \beta$. *Now define two functions* $f, g : \mathbb{Z}^+ \to \mathbb{R}$ *by* $f(n) = n^\alpha$ *for all* $n \geq 1$ *and* $g(n) = n^\beta$ *for all* $n \geq 1$. *Then* $f = O(g)$.

4. Let $f(n) = 4^n$ for all $n \geq 1$. Let $g(n) = 2^n$ for all $n \geq 1$. Using a limit, prove that f is *not* $O(g)$.

5. Let $f(n) = n \log_2 n$ for all $n \geq 1$. Let $g(n) = n$ for all $n \geq 1$. Use a limit to prove that f is *not* $O(g)$. (Notice that the log is base 2.)

6. a) Let $f(n) = \log_2^3(n)$ and let $g(n) = \log_e n^3$. Is $f = O(g)$? Prove your answer.

   b) Now suppose that $h(n) = \log_2 n$ and $g(n) = \log_e n^3$ as before. Is $h = O(g)$? Is $g = O(h)$? Prove both your answers.

1