CS 1050 - Proofs
Homework 8
Assigned Thursday, October 14
Due Thursday, October 21

1. Suppose two players are playing a game with raisins. They start with two non-empty piles of raisins on the table. A move consists of eating all the raisins in one pine and dividing the other pile into two (not necessarily equal) non-empty piles. They take turns until one of them can't move any more; that player loses. Show that if at least one of the piles at the beginning of the game has an even number of raisins, then the first player will win (assuming (s)he knows how to play).

2. Find a formula for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n}$$

by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

3. Find a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)}$$

by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

4. Let $a_n = 3a_{n-1}$ and let $a_0 = 2$.

   a) Write down the first 4 values of the sequence (i.e., $a_0, ..., a_3$).

   b) Guess what you think the solution to the recurrence is (i.e., $a_n =$ what?)

   c) Use induction to prove that your answer is correct.

5. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n \in \mathbb{Z}$, $n > 1$. In the following parts we will consider two sets of initial conditions for this recurrence.

a) Use strong induction to prove that if $a_0 = 0$ and $a_1 = 1$ then $a_n = n$ is the solution to the recurrence relation.

b) Now, assume that for the same recurrence relation the initial conditions are instead $a_0 = a_1 = 5$. Prove that in this case $a_n = 5$ is the solution to the recurrence relation.