CS 1050b - How to Do Proofs - Fall 2004
Practice Midterm 1

1. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ by $f(x_1, x_2) = (2x_1 + x_2, 3x_1 - x_2, 2x_1 + x_2)$ for all reals x_1, x_2.

   Prove that f is one-to-one.

2. Prove the following theorem (showing a Venn diagram is not enough!)

   Theorem 1 *Let A, B, C be any sets. Then*

   $$[(A \cap B) = C] \;\Rightarrow\; [(A \cup C) = A].$$

3. Prove that the sum of 3 consecutive integers is divisible by 3.

4.a) Prove this theorem:

   Theorem 2 $\forall x \in \mathbb{R}, \; \exists y \in \mathbb{R} \; [x^2 = y - 1]$.

   b) Give a counterexample which disproves the following conjecture when the quantifiers are switched:

   Conjecture 1 $\exists y \in \mathbb{R} \; \forall x \in \mathbb{R} \; [x^2 = y - 1]$.

5. Prove that $n^4 - n^2$ is divisible by 3 for all $n \in \mathbb{N}$.