

1 Answers to Chapter 2, Odd-numbered Exercises

- 1) 12^6 . Label each letter in the Hawaiian alphabet as $1, 2, \dots, 12$. Then, each string corresponds to an element in

$$\{1, \dots, 12\} \times \{1, \dots, 12\} \times \{1, \dots, 12\} \times \{1, \dots, 12\} \times \{1, \dots, 12\} \times \{1, \dots, 12\}$$

- 3) The number of passwords with Matt's specified placements is $52^3 \cdot 10^3 \cdot 26^2 \cdot 10^2$. The number of strings made from all of the available characters is $(26 + 26 + 10 + 10)^{10} = 72^{10}$. The second answer is approximately 393,884 times larger than Matt's set of passwords.
- 5) $(26^3 - 25^3) \cdot (10^3 - 1)$. The total number of choices for $l_1 l_2 l_3$ is $26^3 - 25^3$ because there are 26^3 total strings and 25^3 of them contain no K . The total number of choices for $d_1 d_2 d_3$ is $10^3 - 1$ because there are 10^3 total strings and 1 of them contains only zeroes.
- 7) $26^5 \cdot (26 - 5) \cdot 10^2 \cdot 9 \cdot 8$. We have 26 choices for l_1, l_3, l_4, l_5, l_6 , but only $26 - 5 = 21$ choices for l_2 . We have $10 \cdot 9 \cdot 8 = P(10, 3)$ choices for d_1, d_2, d_3 since they form a permutation of 3 letters on the alphabet $\{0, 1, \dots, 9\}$.
- 9) $\binom{17}{3} \cdot 5^3 \cdot 10 \cdot 9 \cdot 8 \cdot (7 + 21)^{14}$. We first choose where we place the vowels. Since they cannot be placed in the last 3 positions, we have 17 available spots from which we choose a subset of size 3. Once the spots for the vowels are chosen, we have 5 choices for which vowel is placed in each of the 3 positions. Then, we place 3 distinct decimal digits at the end of which there are $P(10, 3) = 10 \cdot 9 \cdot 8$ such strings. Lastly, we have 14 positions left to fill, and those 14 positions cannot contain vowels or any of the 3 decimal digits chosen previously.
- 11) **a.** 12^6 since each selection is a string of length 6 (the positions correspond to the employee that the donut will go to) from an alphabet of size 12 (the donut flavors).
b. $P(12, 6) = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7$. These are strings of length 6 from an alphabet of size 12 that are also permutations.
c. $\binom{12}{6}$. He is now selecting a subset of 6 donuts (because the donuts must be different) from a set of 12.
- 13) **a.** $P(20, 4) = 20 \cdot 19 \cdot 18 \cdot 17$ since we are considering strings of length 4 (the position in the string corresponds to the trophy) from an alphabet of size 20 (the students in the competition).
b. $\binom{16}{4}$ because we are selecting a subset of 4 students from the set of 16 students who did not win a trophy. The total number of outcomes is $P(20, 4) \cdot \binom{16}{4}$.
- 15) $\binom{22}{3} - \binom{16}{3}$. We are distributing 25 identical objects (the pencils) into 4 distinguishable bins (the four students). We also have restrictions on the number of objects in each bin. There are $\binom{25+4-1-(1+1+4)}{4-1} = \binom{22}{3}$ ways to do this where Ahmed receives at least one, Dieter receives at least one, and Barbara receives at least four. We subtract out the ways in which Casper receives at least six, $\binom{16}{3}$ to get the total number in which Casper receives at most five.
- 17) $\binom{134}{3}$. With the restriction that $x_4 < 17$, $\binom{134}{3} - \binom{117}{3}$ because we are subtracting out the cases where $x_4 \geq 17$. See example 2.19 for a more detailed explanation.
- 19) **a.** $\binom{471}{65}$.
b. $\binom{471}{65} - \binom{464}{65}$.

- 21) Both sides count the number of ways to choose k people from a group of $m + w$ people. The right hand side counts it directly since it is the number of ways to choose a k -element subset from a set of $m + w$ elements. The left hand side counts it by dividing the $m + w$ people into two groups: m men and w women. It then counts the number of ways to choose j men and $k - j$ women (which is k people). Doing this for all j is mutually exclusive, so we may sum up all these scenarios to count the total number of ways to choose k people from this group.
- 23) $\binom{14}{7}$. We must walk 14 steps, 7 of which must go right, and 7 of which must go up. It is the same as the number of ways to flip a coin 14 times and get 7 heads and 7 tails.
- 25) $\binom{13}{6} \cdot \binom{8}{5} \cdot \binom{8}{3}$. There are $\binom{13}{6}$ paths between the first two points, $\binom{8}{5}$ paths between the second point and the third point, and $\binom{8}{3}$ paths between the third point and the last point.
- 27) $\binom{10}{4} - \binom{6}{3} \binom{4}{3}$. There are $\binom{10}{4}$ paths from the bank to the hideout. Of those paths, $\binom{6}{3} \binom{4}{3}$ go through the police officer. Subtracting these bad paths out gets us the answer.
- 29) The coefficient of $x^{15}y^{120}z^{25}$ is $\binom{100}{15,60,25} \cdot 2^{15} \cdot 3^{60}$. To see this, use the Multinomial Theorem to find the multinomial coefficient of $a^{15}b^{60}c^{25}$ in the expansion of $(a + b + c)^{100}$, then substitute $a = 2x$, $b = 3y^2$, and $c = z$ to get the answer.
- 31) **a.** $\frac{18!}{4!4!2!2!2!1!1!}$. There are $18!$ permutations of length 18. However, since the letter S is repeated 4 times, we must divide by the number of permutations of length 4, and so on. Doing this for each letter in the word results in the answer.
- b.** $\frac{28!}{7!3!3!2!2!2!2!2!1!1!1!}$
- c.** $\frac{27!}{7!3!2!2!2!2!1!1!1!1!1!1!}$