

1 Answers to Chapter 6, Odd-numbered Exercises

1) 2^{2^1} . The total number of relations on $X \times X$ would be 2^{36} since a relation is a subset of $X \times X$ which has size $|X \times X| = 36$. If one only wants symmetric relations however, this is the total number of subsets of $Y := \{\{x, y\} : x, y \in X\} \cup \{(x, x) : x \in X\}$. There are $\binom{6}{2}$ total pairs in the first set, and 6 total pairs in the second set, and hence there are $2^{|Y|} = 2^{15+6}$ total subsets of Y .

The total number of reflexive relations is 2^{15} since we automatically include $\{(x, x) : x \in X\}$ in our relation.

3) No. $(5, 5)$ must be in P in order for P to be reflexive. P is antisymmetric and transitive, so that is the only pair that needs to be added to P .

5) The width and height are the same in P^d as in P . P^d is the dual to P where $(x, y) \in P$ if and only if $(y, x) \in P^d$. The length of the longest chain in P is also the length of the longest chain in P^d since a chain in P is a chain in P^d with orders reversed. The length of the longest antichain in P is the length of the longest antichain in P^d since an antichain in P is an antichain in P^d .

9) $h = 9$. The set of antichains generated by the algorithm are as follows:

$$A_0 = \{12, 16, 18, 22, 23\}$$

$$A_1 = \{2, 3, 11, 13, 17, 21\}$$

$$A_2 = \{4, 10, 25\}$$

$$A_3 = \{5, 8, 24\}$$

$$A_4 = \{20\}$$

$$A_5 = \{9, 19\}$$

$$A_6 = \{6, 7\}$$

$$A_7 = \{1, 26\}$$

$$A_8 = \{14, 15\}$$

11) $\binom{10}{5}$. One may view the set of different menus as the set of all subsets of a set of size 10 (the set of main course dishes). The largest set of different menus is equal to the size of the largest antichain in this subset lattice. The largest antichain is the width of the poset, and by Sperner's theorem (Theorem 6.15), this is $\binom{10}{5}$ for the subset lattice.

13) .

15) It does not have an interval order since $\{7, 1\}$ and $\{8, 4\}$ form an induced copy of $\mathbf{2+2}$.

17) It does not have an interval order since $\{9, 6\}$ and $\{4, 11\}$ form an induced copy of $\mathbf{2+2}$.

19) We first prove that $|\mathcal{D}| = |\mathcal{U}|$.

21) .