Student Name and ID Number

MATH 3012 Final Exam, December 8, 2004, WTT

1. Consider the family of all strings of length 27 formed from the eight letter alphabet $\{A, B, C, D, E, F, G, H\}$.

a. What is the total number of strings?

b. How many strings have exactly six A's?

c. How many strings have exactly six A's, five B's, seven C's and nine G's?

d. Of the strings described in part c, how many have all five B's before the nine G's?

e. Of the strings described in part d, how many have the five B's and the nine G's occuring together as a block of fourteen consecutive characters?

2. How many integer value solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 = 45$, all $x_i > 0$.

b. $x_1 + x_2 + x_3 = 45$, all $x_i \geq 0$.

c. $x_1 + x_2 + x_3 < 45$, all $x_i \geq 0$.

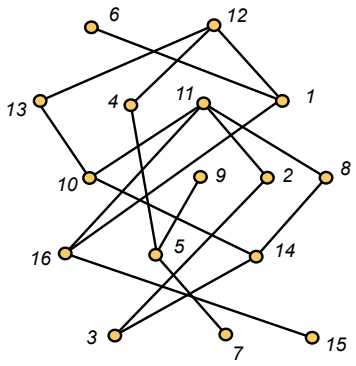
d. $x_1 + x_2 + x_3 \leq 45$, all $x_i \geq 0$.

e. $x_1 + x_2 + x_3 = 45$, $x_1, x_2 > 0$, $x_3 > 8$.

3. Use the Euclidean algorithm to find $d = \gcd(1260, 336)$.

Then find integers x and y so that $d = 1260x + 336y$.

4. Consider the partially ordered set (poset) shown below on the left:



a. Find the set of maximal elements.

b. Find the height h of this poset and find a partition into h antichains.

c. Find a chain of h points in this poset.

5. Compute $\phi(360)$ (Hint: Factor 360 into a product of primes).

6 a. Write all the partitions of the integer 7;

b. Of the partitions listed in part a, how many use distinct parts?

c. Of the partitions listed in part a, how many use odd parts?

7. Write the inclusion/exclusion formula for the number of onto functions from $\{1, 2, \ldots, m\}$ to $\{1, 2, \ldots, n\}$.

8. Write the inclusion/exclusion formula for the number of derangements of $\{1, 2, \ldots, n\}$.

9. Let A denote the advancement operator, i.e., $Af(n) = f(n+1)$. Find the general solution of the following equation:

$$(A^2 + 2A - 35)f(n) = 0$$

10. For the equation in the preceding problem, find the particular solution given $f(0) = 5$ and $f(1) = 1$.

11. Find the general solution of the following equation:

$$(A-2)^3(A+3)^2(A-3+2i)^4 f(n) = 0$$

12. Let r_n denote the number of regions in the plane determined by n lines—given that each pair of lines intersects in a point and that no point in the plane belongs to three or more lines.
a. Write a recurrence equation for r_n.

b. Solve the recurrence equation in part (a).

13. The following text file lists the weights on the edges of a graph whose vertex set is $\{1, 2, \ldots, 7\}$. The lines in the file have been sorted so that the weights are non-decreasing. In the middle column, list the edges identified by Kruskal's Agorithm (Avoid Cycles) as belonging to a minimum weight spanning tree. Then in the far right column, list the edges in the order they would identified by Prim's Algorithm (Build Tree), using vertex 6 as the root.

`graphdata.txt`	**Kruskal**	**Prim**
1 2 20		
3 7 24		
4 6 26		
1 7 27		
2 3 27		
2 4 30		
1 6 32		
5 7 33		
3 5 34		
2 7 35		
1 3 36		

14. Consider the graph G whose edges are listed in the text file above.

a. Show that G is planar by providing a drawing in the plane with no crossing edges.

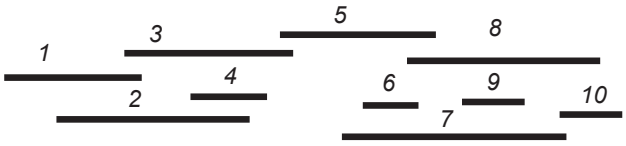
b. Show that G has an Eulerian circuit by providing a listing of vertices so that (1) the sequence starts and ends with 1, (2) each consecutive pair of vertices in the list forms an edge, and (2) each edge occurs as a consecutive pair exactly once.

c. Show that G has a hamiltonian cycle by listing the vertices in an order which defines such a cycle.

15. The matrix given is the distance matrix for a oriented graph G whose vertex set is $\{1, 2, 3, 4, 5, 6\}$. In the space to the right, apply Dijkstra's algorithm to find all the shortest paths from vertex 1 to all other vertices in the G.

D	1	2	3	4	5	6
1	0	50	14	13	20	55
2	31	0	28	8	19	7
3	26	33	0	9	3	27
4	16	35	8	0	6	26
5	23	16	7	13	0	15
6	19	12	28	15	28	0

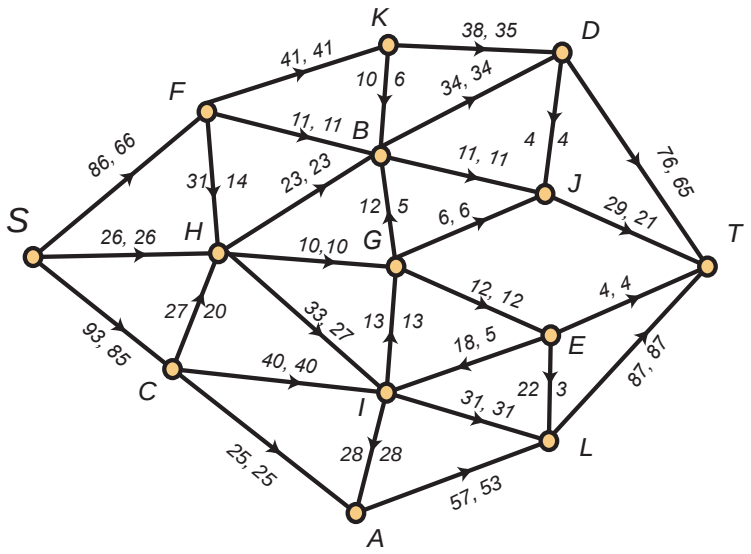
16. The intervals in the following family have been labelled in the order determined by their left end points. Use the First Fit (Greedy) algorithm to determine an optimal coloring of the interval graph G associated with this family of intervals. You may indicate this coloring directly on the figure.



17. Explain why a graph with 1270 vertices and 5983 edges is non-planar.

18. Explain why a graph with 200 vertices and $13,839$ edges contains a triangle.

19. Consider the following network flow:



a. What is the current value of the flow?

b. What is the capacity of the following cut: $L = \{S, B, C, D, F, H, K\}$, $U = \{T, A, E, G, I, J, L\}$.

c. Apply the labeling algorithm and list the vertices and labels in the order determined by preferring source and sink and then taking all remaining vertices in alphabetic order.

d. List the vertices of the augmenting path determined in part a.

e. Update the flow by making the appropriate changes directly on the diagram. What is the new value of the flow?

f. Apply the labeling algorithm to the updated flow. It should halt with the sink unlabeled.

g. Use the results of the labeling algorithm to determine a saturated cut:

$L = \{\text{S},$ $\}$, and $U = \{T,$ $\}$