Student Name and ID Number

MATH 3012 Final Exam, May 4, 2006, WTT

1. All four parts of this problem are concerned with ternary strings of length n, i.e., words of length n with letters from the alphabet $\{0, 1, 2\}$.

a. How many ternary words of length 23?

b. How many ternary words of length 23 with eight 0's, nine 1's and six 2's?

c. Let t_n denote the number of ternary strings that do not have a 1 followed immediately by a 2. Find (but do not solve) a linear recurrence equation satisfied by t_n.

2. How many lattice paths from $(2, 3)$ to $(17, 12)$ pass through $(4, 6)$ and $(8, 10)$?

3. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 + x_4 = 40$, all $x_i > 0$.

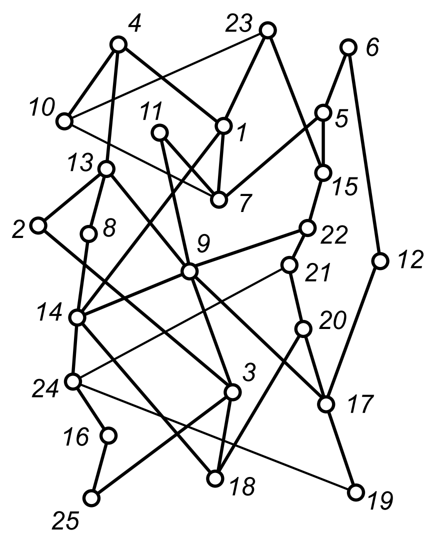
b. $x_1 + x_2 + x_3 + x_4 = 40$, all $x_i \geq 0$.

c. $x_1 + x_2 + x_3 + x_4 \leq 40$, all $x_i \geq 0$.

4. Use the Euclidean algorithm to find $d = \gcd(168, 1320)$.

5. Use your work in the preceding problem to find integers x and y so that $d = 168x + 1320y$.

6.



a. Find the set of minimal elements of this poset.

b. How many elements of are incomparable with the point labeled 12?

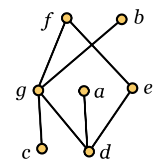
c. Explain why $\{3, 16, 17\}$ is not a maximal antichain.

d. For each x, let height(x) denote the maximum size of a chain having x as its greatest element. Writing directly on the diagram, label each point with the integer representing its height.

e. Find the height h of this poset

f. Find a chain of h points.

7.

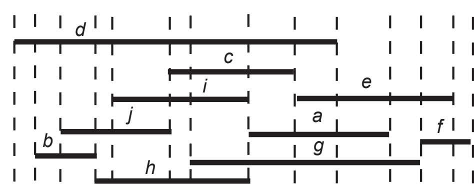


a. This poset is an interval order and has 5 distinct down sets. Find them.

b. This poset also has 5 distinct up sets. Find them.

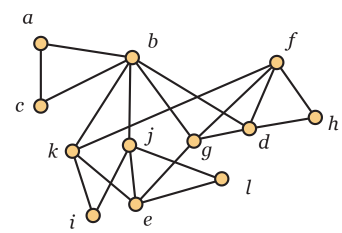
c. Find the unique interval representation for this poset where every element is assigned an interval with integer endpoints from $\{1, 2, 3, 4, 5\}$.

8. Define an interval order P with point set $X = \{a, b, c, d, e, f, g, h, i, j\}$. by the following interval representation.



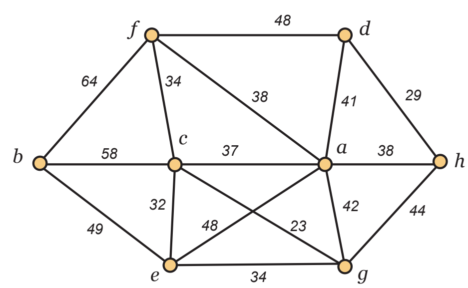
Use the First Fit algorithm to a partition of this poset into a minimum number of chains. Provide your answer by labeling the intervals in the diagram with positive integers so that all elements assigned the same integer form a chain. Then find a maximum antichain in this poset.

9.



Use the Greedy Algorithm and alphabetic order to find an euler circuit in the graph above. Your answer should be given as a sequence of partial circuits starting with the trivial circuit (a).

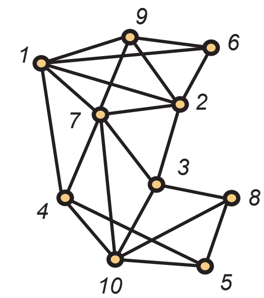
10.



In the space below, list *in order* the edges which make up a minimum weight spanning tree using, respectively Kruskal's Algorithm (avoid cycles) and Prim's Algorithm (build tree). For Prim, use vertex a as the root.

Kruskal's Algorithm **Prim's Algorithm**

11.



a. Show that this graph is hamiltonian by listing the vertices in an order which forms a cycle of size 10.

b. Explain why this graph has neither an euler circuit nor an euler path.

12. A data file `digraph_data.txt` has been read for a digraph whose vertex set is [6]. The weights on the directed edges are shown in the matrix below. Apply Dijkstra's algorithm to find the distance from vertex 1 to all other vertices in the graph. Also, for each x, find a shortest path from 1 to x.

W	1	2	3	4	5	6
1	0	13	51	20	34	7
2	60	0	28	9	19	8
3	46	60	0	19	9	60
4	16	43	17	0	8	14
5	23	11	7	13	0	28
6	19	8	82	16	28	0

13. Write the general solution of the advancement operator equation: $(A-2)^3(A-1)^4(A+6)^2(A-8)f = 0$.

14. Find a particular solution to the advancement operator equation: $(A^2 - 9A + 18)f(n) = 20(2)^n$.

15. Find the unique solution to the advancement operator equation: $(A^2 - 9A + 18)f(n) = 20(2)^n$ with $f(0) = 3$ and $f(1) = 16$.

16. Let X be a set and let $\mathcal{P} = \{P_1, P_2, \ldots, P_m\}$ be a family of properties. For each subset $S \subseteq \{1, 2, \ldots, m\}$, let $N(S)$ denote the number of elements of X which satisfy property P_i whenever $i \in S$. Write the Inclusion-Exclusion formula for the number of elements of X which satisfy none of the properties in $\mathcal{P}$:

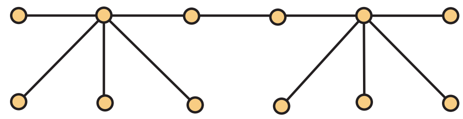
17. Write the Inclusion-Exclusion formula for the Euler-ϕ function.

18. Use the formula from the preceding problem to find $\phi(n)$ when $n = 2^4 \times 3^2 \times 5^3$.

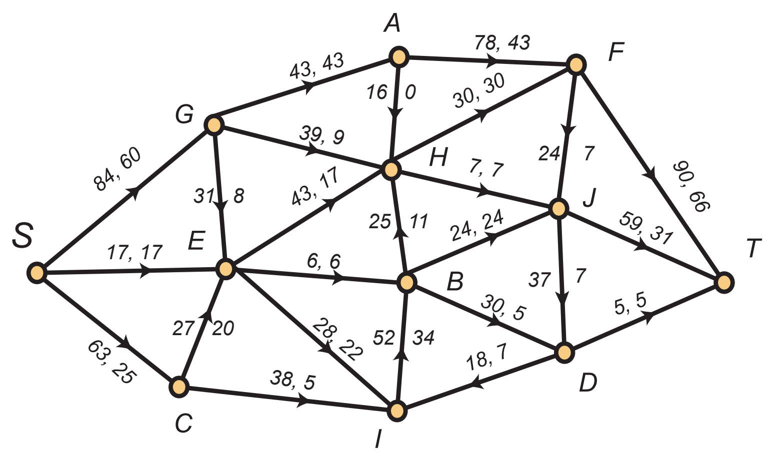
19. Let $R(n, m)$ denote the least positive integer t so that every graph on t vertices contains a complete subgraph of size n or and independent set of size m. Bob claims that $R(3, 3) = R(4, 4) = 6$. Alice replies that Bob is only half right. $R(3, 3) = 6$ but $R(4, 4) > 6$. Explain why Alice's assertion that $R(4, 4) > 6$ is correct.

20. What is the formula for the number of labeled trees with vertex set $\{1, 2, \ldots, n\}$?

21. How many ways are there to assign labels from the set $\{1, 2, \ldots, 10\}$ to the unlabeled tree shown below?



22.



a. What is the current value of the flow?

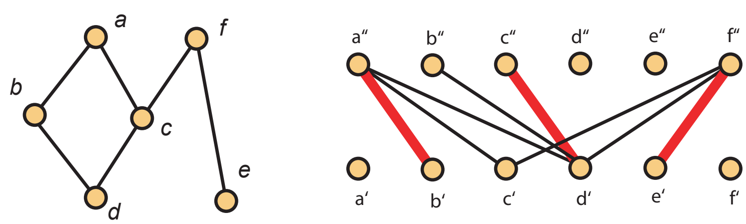
b. What is the capacity of the cut $V = \{S, A, C, I, E, G, H\} \cup \{B, D, F, J, T\}$.

c. Carry out the labeling algorithm, using the pseudo-alphabetic order on the vertices and list below the labels which will be given to the vertices.

d. Use your work in part c to find an augmenting path and make the appropriate changes directly on the diagram.

e. Carry out the labeling algorithm a second time on the updated flow. It should halt without the sink being labeled. Find a cut whose capacity is equal to the value of the flow.

23.



In the figure above, we show a poset and the bipartite graph associated with it. The darkened edges form a maximum matching in the graph. Find the minimum chain partition determined by this matching.