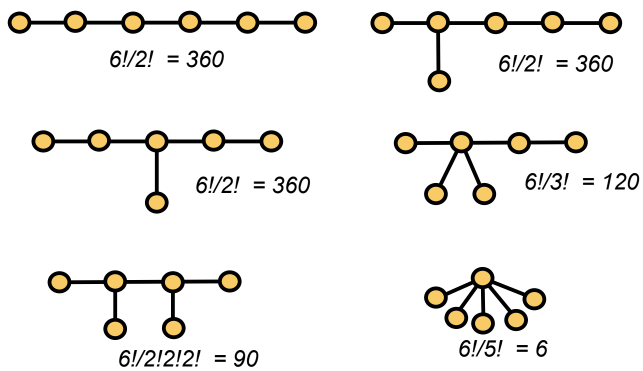
Student Name and ID Number

**MATH 3012 Quiz 3, November 23, 2004, WTT**

**1.** Draw all the unlabeled trees on 6 vertices. Then for each, count the number of ways the vertices can be labeled with the integers from $\{1, 2, \ldots, 6\}$.




*Total = 360 + 360 + 360 + 120 + 90 + 6 = 1296*

**2.** The following text file lists the weights on the edges of a graph whose vertex set is $\{1, 2, \ldots, 9\}$. The lines in the file have been sorted so that the weights are non-decreasing. In the middle column, list the edges identified by Kruskal's Agorithm (Avoid Cycles) as belonging to a minimum weight spanning tree. Then in the far right column, list the edges in the order they would identified by Prim's Algorithm (Build Tree), using vertex 8 as the root.

| `graphdata.txt` | **Kruskal** | **Prim** |
|---|---|---|
| 2 7 18 | 2 7 18 | 2 8 35 |
| 1 4 22 | 1 4 22 | 2 7 18 |
| 4 9 24 | 4 9 24 | 2 3 28 |
| 4 5 24 | 4 5 24 | 1 2 32 |
| 1 9 26 | 2 3 28 | 1 4 22 |
| 2 3 28 | 1 6 30 | 4 9 24 |
| 1 6 30 | 1 2 32 | 4 5 24 |
| 6 9 31 | 2 8 35 | 1 6 30 |
| 1 2 32 | | |
| 3 6 32 | | |
| 5 7 33 | | |
| 2 8 35 | | |
| 7 8 37 | | |
| 7 9 37 | | |

**Note:** Both trees have total weight $213 = 18 + 22 + 24 + 24 + 28 + 30 + 32 + 35$.

**3.** Consider the graph $G$ whose edges are listed in the preceding text file.

**a.** Explain why $G$ does not have an Eulerian circuit.

A graph has an Euler circuit if and only if it is connected and every vertex has even degree. The graph is connected since it has a spanning tree. However, vertices 4 and 6 have odd degree (each of these two vertices has degree 3).

**b.** Show that $G$ has an Eulerian path by providing a listing of vertices so that (1) each consecutive pair of vertices in the list forms an edge, and (2) each edge occurs as a consecutive pair exactly once.

$$6, 3, 2, 8, 7, 2, 1, 4, 9, 1, 6, 9, 7, 5, 4$$

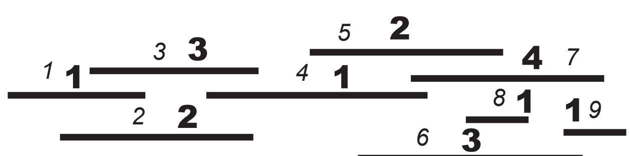
**c.** Show that $G$ has a hamiltonian cycle by listing the vertices in an order which defines such a cycle.

$$1, 6, 3, 2, 8, 7, 5, 4, 9$$

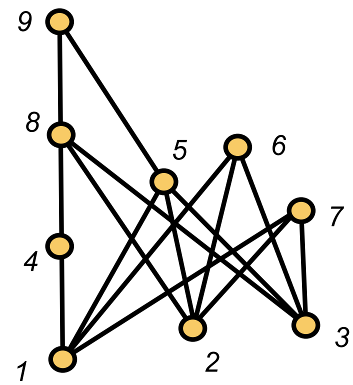
**4.** The matrix given is the distance matrix for a oriented graph $G$ whose vertex set is $\{1, 2, 3, 4, 5, 6, 7\}$. In the space to the right, apply Dijkstra's algorithm to find all the shortest paths from vertex 1 to all other vertices in the $G$.

| D | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | Scan | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 12 | 51 | 20 | 34 | 5 | 19 | | 1 | 0 | 12 | 51 | 20 | 34 | 5 | 19 |
| | | | | | | | | | | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 |
| 2 | 80 | 0 | 28 | 8 | 19 | 7 | 24 | | 6 | 0 | 11 | 51 | 20 | 34 | 5 | 15 |
| | | | | | | | | | | 1,1 | 1,6,2 | 1,3 | 1,4 | 1,6,5 | 1,6 | 1,6,7 |
| 3 | 46 | 60 | 0 | 19 | 9 | 60 | 80 | | 2 | 0 | 11 | 39 | 19 | 30 | 5 | 15 |
| | | | | | | | | | | 1,1 | 1,6,2 | 1,6,2,3 | 1,6,2,4 | 1,6,2,5 | 1,6 | 1,6,7 |
| 4 | 16 | 43 | 17 | 0 | 8 | 14 | 19 | | 7 | 0 | 11 | 38 | 19 | 30 | 5 | 15 |
| | | | | | | | | | | 1,1 | 1,6,2 | 1,6,7,3 | 1,6,2,4 | 1,6,2,5 | 1,6 | 1,6,7 |
| 5 | 23 | 11 | 7 | 13 | 0 | 28 | 22 | | 4 | 0 | 11 | 38 | 19 | 27 | 5 | 15 |
| | | | | | | | | | | 1,1 | 1,6,2 | 1,6,7,3 | 1,6,2,4 | 1,6,2,4,5 | 1,6 | 1,6,7 |
| 6 | 19 | 6 | 82 | 15 | 28 | 0 | 10 | | 5 | 0 | 11 | 37 | 19 | 27 | 5 | 15 |
| | | | | | | | | | | 1,1 | 1,6,2 | 1,6,2,4,5,3 | 1,6,2,4 | 1,6,2,4,5 | 1,6 | 1,6,7 |
| 7 | 11 | 16 | 23 | 25 | 19 | 8 | 0 | | | | | | | | | |

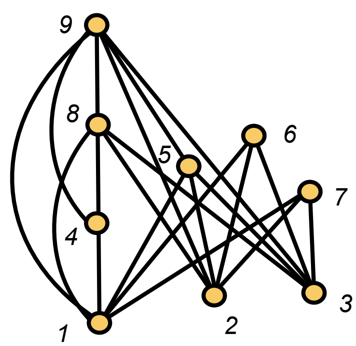
**5.** The intervals in the following family have been labelled in the order determined by their left end points. Use the First Fit (Greedy) algorithm to determine an optimal coloring of the associated interval graph. You may indicate this coloring directly on the figure.



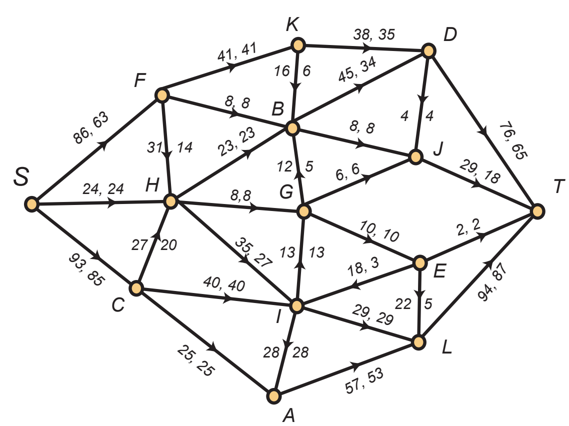
**6.** Draw the poset diagram for the interval order associated with the family in Problem 5.




**7.** Draw the comparability graph of the poset in Problem 6.




**8.** Consider the following network flow:




**a.** What is the current value of the flow?

$$63 + 24 + 85 = 172$$

**b.** What is the capacity of the following cut: $L = \{S, B, C, D, F, H, K\}$, $U = \{T, A, E, G, I, J, L\}$.

$$76 + 4 + 8 + 8 + 35 + 40 + 25 = 196$$

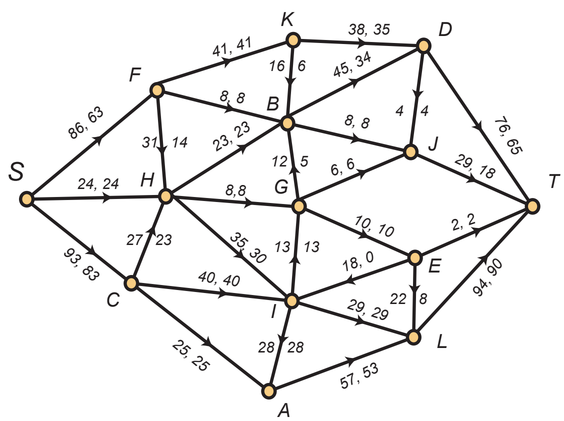
**c.** Apply the labeling algorithm and list the vertices and labels in the order determined by preferring source and sink and then taking all remaining vertices in alphabetic order.

$S \quad (*, +, \infty)$
$C \quad (S, +, 8)$
$F \quad (S, +, 23)$
$H \quad (C, +, 7)$
$I \quad (H, +, 7)$
$E \quad (I, -, 3)$
$G \quad (E, -, 3)$
$L \quad (E, +, 3)$
$B \quad (G, +, 3)$
$T \quad (L, +, 3)$

**d.** List the vertices of the augmenting path determined in part a.

$$S, C, H, I, E, L, T$$

**e.** Update the flow by making the appropriate changes directly on the diagram. What is the new value of the flow?




The new value of the flow is $172 + 3 = 175$.

**f.** Apply the labeling algorithm to the updated flow. It should halt with the sink unlabeled.

$S \quad (*, +, \infty)$
$C \quad (S, +, 5)$
$F \quad (S, +, 23)$
$H \quad (C, +, 4)$
$I \quad (H, +, 4)$

**g.** Use the results of the labeling algorithm to determine a saturated cut:

$$L = \{S, C, F, H, I\} \quad U = \{T, A, B, D, E, G, J, K\}$$

Note that the capacity of this cut is

$$41 + 8 + 23 + 8 + 13 + 29 + 28 + 25 = 175$$