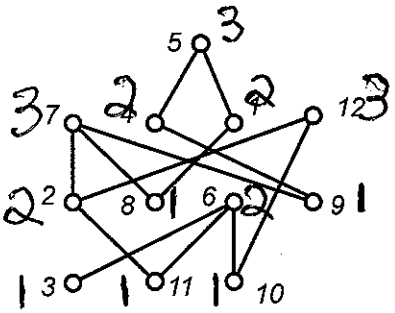
KEY

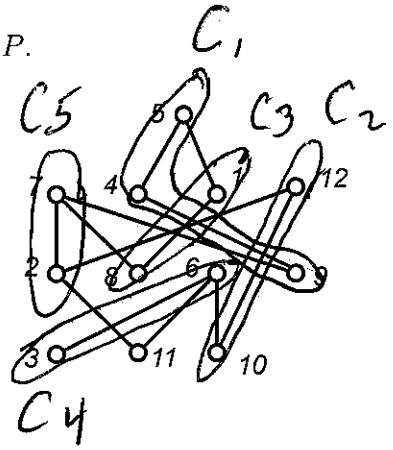
Student Name and ID Number

MATH 3012 Quiz 2, October 22, 2010, WTT

36 pts
9 x 4

1. Shown below are two identical copies of a poset P.





a. List the maximal elements of P.

5, 6, 7, 12

b. List the elements which are comparable with 9 in P.

4, 5, 7

c. List the elements which cover 9 in P.

4, 7

d. List elements which form a maximal chain of size 2.

possible answers: {7,8}, {3,6}, {6,11}, {6,10}, {7,9}, {10,12}

e. List elements which form a maximal chain of size 3.

{1,5,8}, {4,5,9}, {2,11,12}, {2,7,11} + others

f. List elements which form a maximal antichain of size 4.

{5,6,7,12}, {2,6,8,9}, and others

g. Find the height h of P; list here h points from P that form a maximum chain; and using the left copy of P, indicate a partition of P into h antichains.

e.g. {5,4,9} $h = 3$

h. Find the width w of P; list here w points from P that form a maximum antichain; and using the right copy of P, indicate a partition of P into w chains.

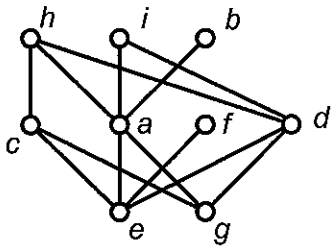
$w = 5$ $A = \{1, 4, 6, 7, 12\}$

i. Show that P is not an interval order by finding four points that form a copy of $\mathbf{2+2}$. Hint: Do this by inspection. Don't carry out the entire algorithm for finding representations.

{2, 11, 4, 9} e.g.

2. Consider the interval order Q shown below.

30 pts
n+10



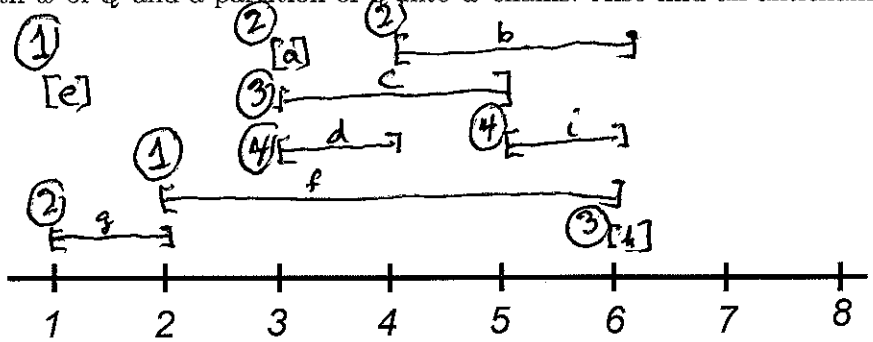
a. For each element x of Q, find the down set $D(x)$ and the upset $U(x)$.

$D(a) =$	eg	3	$U(a) =$	hib	3
$D(b) =$	aeg	4	$U(b) =$	$\emptyset$	6
$D(c) =$	eg	3	$U(c) =$	h	5
$D(d) =$	eg	3	$U(d) =$	hi	4
$D(e) =$	$\emptyset$	1	$U(e) =$	hib acfd	1
$D(f) =$	e	2	$U(f) =$	$\emptyset$	6
$D(g) =$	$\emptyset$	1	$U(g) =$	hib acd	2
$D(h) =$	acdeg	6	$U(h) =$	$\emptyset$	6
$D(i) =$	adeg	5	$U(i) =$	$\emptyset$	6

b. If m is the number of distinct down sets, find the unique interval representation of Q using intervals with integer endpoints from $\{1, 2, \ldots, m\}$.

$I(a) =$	[3,3]	$I(b) =$	[4,6]
$I(c) =$	[3,5]	$I(d) =$	[3,4]
$I(e) =$	[1]	$I(f) =$	[2,6]
$I(g) =$	[1,2]	$I(h) =$	[6,6]
$I(i) =$	[5,6]		

c. Draw the interval representation for Q obtained in part b. Then use the First Fit Algorithm to find the width w of Q and a partition of Q into w chains. Also find an antichain of size w.



Maximum antichains: a, c, d, f
b, e, f, i

20 pts
4×5

3. Let 2^{14} be the poset consisting of all subsets of $\{1, 2, 3, \ldots, 14\}$, ordered by inclusion.

a. What is the height of this poset?

15

b. What is the width of this poset?

$\binom{14}{7}$

c. How many maximal chains does the poset have?

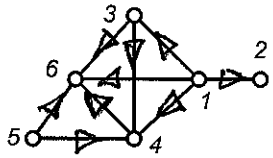
14!

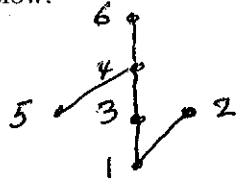
d. How many maximal chains in this poset pass through the set $\{2, 4, 8, 11, 13\}$?

5! 9!

15 pts
3×5

4. Consider the graph G shown below.

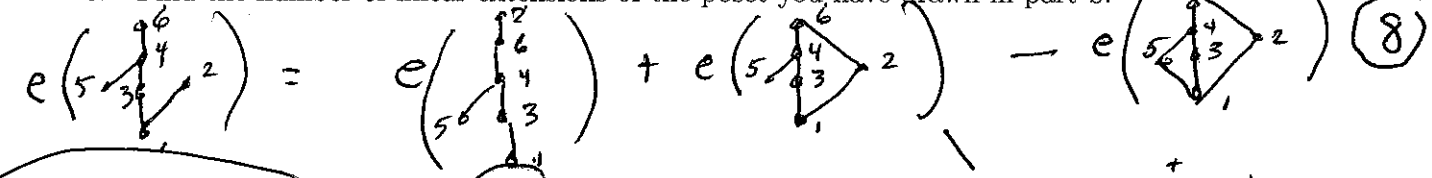


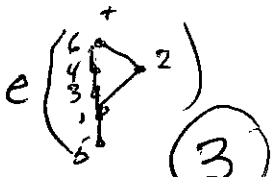


a. Show that G is a comparability graph by orienting the edges transitively. Show your work directly on the figure.

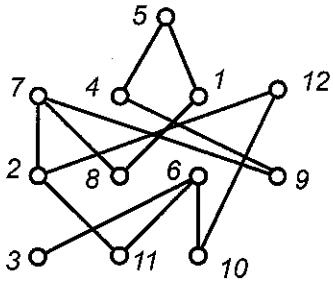
b. In the space to the right of the figure, draw the diagram of the poset associated with your answer to part a. Four possible answers. Order of 4 and 6 in my picture can be reversed. While thing can be inverted.

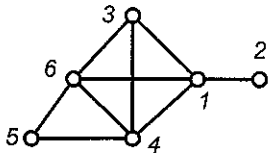
c. Find the number of linear extensions of the poset you have drawn in part b.





Duplicate Figures





Grading Summary

#1. (36) = 9 × 4

#2. (30) = 3 × 10

#3. (20) = 4 × 5

#4 (15) = 3 × 5

Perfect paper sets

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