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Student Name and ID Number

MATH 3012 Quiz 1, September 27, 2011, WTT

1. Consider the 16-element set consisting of the ten digits $\{0, 1, 2, \ldots, 9\}$ and the six capital letters $\{A, B, C, D, E, F\}$.

a. How many strings of length 9 can be formed if repetition of symbols is *not* permitted?

b. How many strings of length 9 can be formed if repetition of symbols is permitted?

c. How many strings of length 9 can be formed using exactly two 6's, three B's and four D's?

d. How many strings of length 9 can be formed using exactly two 6's, three B's and four D's if the four $D's$ are required to occur consecutively in the string?

2. How many lattice paths from $(2, 8)$ to $(27, 39)$ do *not* pass through $(18, 23)$?

3. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 + x_4 = 32$, all $x_i > 0$.

b. $x_1 + x_2 + x_3 + x_4 = 32$, all $x_i \geq 0$.

c. $x_1 + x_2 + x_3 + x_4 < 32$, all $x_i > 0$.

d. $x_1 + x_2 + x_3 + x_4 \leq 32$, all $x_i \geq 0$.

e. $x_1 + x_2 + x_3 + x_4 = 32$, all $x_i > 0$, $x_2 \geq 8$.

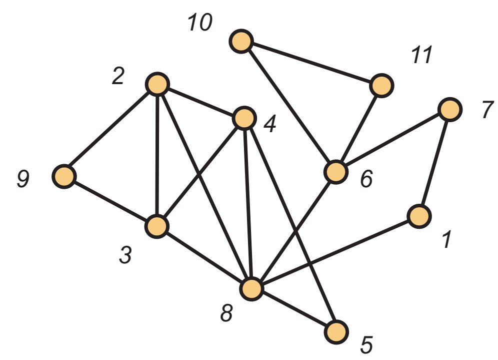
f. $x_1 + x_2 + x_3 + x_4 = 32$, all $x_i > 0$, $x_2 \leq 13$.

4. Use the Euclidean algorithm to find $d = \gcd(630, 495)$.

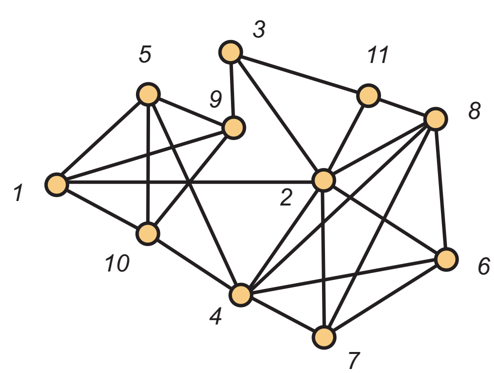
5. Use your work in the preceding problem to find integers a and b so that $d = 630a + 495b$.

6. For a positive integer n, let s_n count the number of ternary strings of length n that do not contain 00 or 01 as a substring. Note that $s_1 = 3$ and $s_2 = 7$. Develop a recurrence relation for s_n and use it to compute s_3, s_4 and s_5.

7. Use the algorithm developed in class, with vertex 1 as root, to find an Euler circuit in the following graph:



8. Consider the following graph:



a. Explain why this graph does not have an Euler circuit.

b. Provide a listing of the vertices that constitutes a Hamiltonian cycle *starting* with vertices 1, 2 and 3 in that order.

c. Find a set of vertices that forms a maximal clique but not a maximum clique.

d. What is $\omega(G)$ for this graph?

e. Find a set of vertices which forms a maximum clique in this graph.

f. Show that $\chi(G) = \omega(G)$ for this graph by providing an optimum coloring. You may write directly on the figure.

9. Draw a graph G on six vertices with $\omega(G) = 3$ and $\chi(G) = 4$.

10. Draw all unlabelled trees on five vertices. Then for each of them, count the number of ways the labels from $\{1, 2, 3, 4, 5\}$ can be applied. Hint: The total number of labeled trees on 5 vertices is $125 = 5^3$.

11. Prove the following identity by Mathematical Induction:

$$2 + 8 + 14 + \ldots 6n - 2 = 3n^2 - n \quad \text{when } n \geq 1.$$