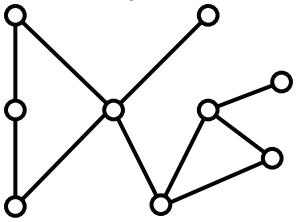
Student Name and ID Number

MATH 3012 Quiz 2, October 25, 2011, WTT

1. Dave claims there is a triangle-free graph with 38 vertices and 412 edges. Yolanda says he is wrong. Explain who is right.

2. Zori claims that there is planar graph with 1012 vertices and 3672 edges. Carlos says she is wrong. Explain who is right.

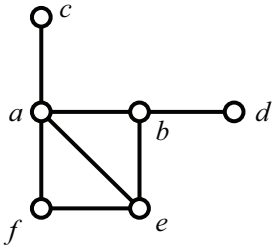
3. Verify Euler's formula for this planar graph.



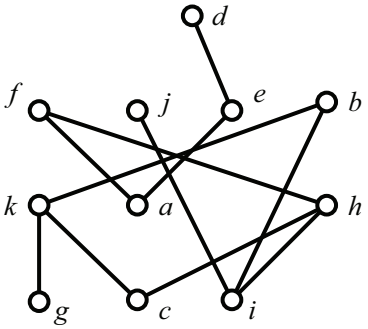
4. How many symmetric binary relations are there on $\{1, 2, \ldots, n\}$? Of these how many are reflexive?

5. a. Let $X = \{a, b, c, d, e\}$ and let $P = \{(a, a), (b, b), (c, c), (d, d), (e, e), (d, b), (d, c), (a, e), (a, b), (e, b)\}$. Draw a diagram for the poset (X, P).

6. Show that the following graph is a comparability graph by transitively orienting the edges. In the space to the right, draw the diagram of the associated poset.



7. Consider the following poset.



a. Find all points comparable to a. _____

b. Find all points which cover a. _____

c. Find a maximal chain of size 2. _____

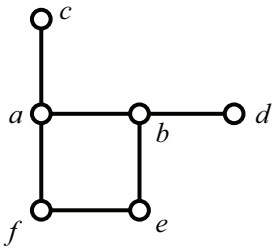
d. Find a maximal antichain of size 3. _____

e. Find the set of all minimal elements. _____

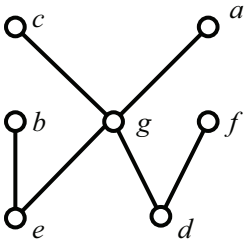
f. Using the algorithm taught in class (recursively removing the set of minimal elements), find the height h of the poset and a partition of P into h antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam.

The height h is _____ and _____ is a maximum chain.

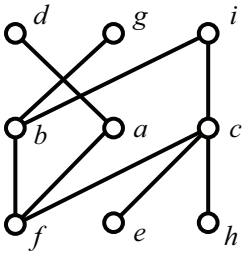
8. In the space to the right, draw the diagrams of three different posets all having the following graph as their cover graphs (a) a height 2, width 3 poset; (b) a height 3, width 3 poset; and (c) a height 4, width 2 poset.



9. By inspection (not by algorithm), find four points in the following poset that form a $\mathbf{2+2}$.

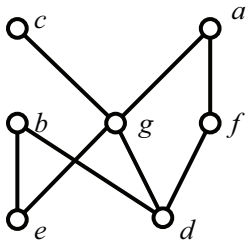


10. Find by inspection the width w of the following poset and find a partition of the poset into w chains. Also find a maximum antichain. You may indicate the partition by writing directly on the diagram.



The width w is _____ and _____ is a maximum antichain.

11. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation. Then use the First Fit coloring algorithm to find the width w and a partition of the poset into w chains. Also, find a maximum antichain.



The width w is _____ and _____ is a maximum antichain.

12. Let $\mathbf{2}^{13}$ be the poset consisting of all subsets of $\{1, 2, 3, \ldots, 13\}$, ordered by inclusion.

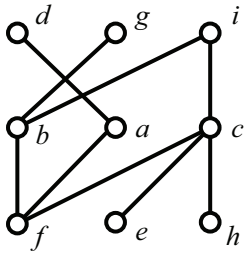
a. What is the height of this poset?

b. What is the width of this poset?

c. How many maximal chains does the poset have?

d. How many maximal chains in this poset pass through the set $\{2, 5, 8, 10\}$?

13. Extra Credit. Show that the poset shown below has dimension 2 by finding two linear extensions whose intersection is the partial order.



Duplicate Figures