Yucong Liu - LU decomposition and Toeplitz decomposition of a neural network

Any matrix A has an LU decomposition up to a row or column permutation. Less well-known is the fact that it has a ‘Toeplitz decomposition’. We will prove that any continuous function has an approximation to arbitrary accuracy by a neural network, where the weight matrices alternate between lower and upper triangular matrices, and the activation σ may be chosen to be essentially any uniformly continuous nonpolynomial function. The same result also holds with Toeplitz matrices, to arbitrary accuracy, and likewise for Hankel matrices. A consequence of our Toeplitz result is a fixed-width universal approximation theorem for convolutional neural networks, which so far have only arbitrary width versions. Since our results apply in particular to the case when f is a general neural network, we may regard them as LU and Toeplitz decompositions of a neural network. The practical implication of our results is that one may vastly reduce the number of weight parameters in a neural network without sacrificing its power of universal approximation. We will present several experiments on real data sets to show that imposing such structures on the weight matrices dramatically reduces the number of training parameters with almost no noticeable effect on test accuracy.

Emeka Peter Mazi - A Mathematical Model to Investigate the Deposition and Erosion Processes in Porous Media with Branching Internal Structure

Deposition and erosion processes have significant effects in the nurture, industrial applications, and in general porous media. In this work, we study the deposition and erosion of solid particles at the micro-scale level and their direct consequences on the internal structure of porous media with complex internal morphology. We formulate a mathematical model to investigate these phenomena in a porous medium with branching structure consisting of cylindrical channels, undergoing a uni-directional flow. The flow and the transport of solid particles are modeled by Hagen-poiseuille law and advection-diffusion equations, respectively. As a consequence of deposition and erosion, those cylindrical channels tend to shrink or expand, respectively based on some key parameters which are discussed in this paper. We characterize the evolution of the internal morphology of the porous medium by investigating the initial porosity gradient within the porous medium depth. The study observes that high thickness scale leads to uniform thickness with exclusive deposition or erosion, while low thickness scale results in non-uniform thickness with both processes; similarly, high pore scale yields uniform pore radii with erosion dominance, whereas low pore scale causes non-uniform pore sizes and porosity-graded structures favoring deposition over erosion. Finally, we discuss a two-layer asymmetric branching structure with varying pore radii and analyze how the structurebehaves in terms of erosion and deposition.

Yi-Yung Yang - The application of maximum-principle-satisfying method on coupled flow and transport problem

We explored various pairs of finite elements to solve the coupled flow and transport problems in porous media. The talk presents the technique to address the challenges arising from multi-physical issues such as coupling flow and transport in porous media. The preservation of the physical properties, such as the maximum principle for transport and local mass conservation for flow is crucial.