MATH 1501 J3/J4/J5 Final Exam

Fall 2008

Name: _____

GTid (9xxxxxxxx): _____

Instructor: Allen K. Hoffmeyer

Teaching Assistant and Section: _____

There are 11 questions on this exam on 13 pages (not counting this coverpage). Be sure to explain your answers, *as answers that are not accompanied by explanations/work may receive no credit.* You are to complete this exam completely alone, *without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.*

By signing on the line below, you agree to abide by the Georgia Tech Honor Code, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Failure to sign this cover page will *not* be considered evidence of academic misconduct. However, **if the cover page is not signed, seven points may be deducted from your raw total score on this exam.**

Student signature: _____

Question	Points	Score		Question	Points	Score
1	5			6	5	
2	5			7	5	
3	5			8	5	
4	5			9	5	
5	5			10	5	
				Bonus	5	

Total (out of 50):

Average (out of 5):

1. (5 points) Use the definition of a derivative to find $f'(x)$ where $f(x) = \frac{3}{x^2}$ for $x \neq 0$. *Note: Your answer should involve a limit. Finding the derivative without using the definition is worth zero points.*

2. (5 points) Determine the value A for which the function $g(x) = x^2 + \frac{A}{x}$ has a relative minimum at $x = 2$. *Note: To receive full credit on this problem, you need to show that $x = 2$ is a relative minimum with the value of A that you found using either the first derivative test or the second derivative test.*

3. (5 points) Let m and n be positive integers. Find the maximum of the function

$$f(x) = x^m(1-x)^n$$

on the interval $[0, 1]$. *Note: Your answer should only involve m's and n's.*

4. (5 points) Show that the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$. *Note: Your answer must involve limits, e.g. pictures of graphs and intuitive explanations will not receive credit.*

5. (5 points) Find $f'(x)$ for all x in the domain of f where

$$f(x) = \ln\left(\sqrt[3]{\frac{(x+1)(x+2)(x+3)(x+4)}{x^2}}\right) + \int_x^{x^2} \sin\sqrt{t}\, dt + x^{99}e^{\pi x}.$$

Hint: For the first term of f, you should simplify the expression using the properties of logarithms rather than doing it the long way. Also, please notice that the integral is a function of x.

6. (5 points) What conditions must the *constants* a, b, and c satisfy if the cubic polynomial $y = x^3 + ax^2 + bx + c$ is to have a point of inflection with a horizontal tangent line. *Note: The horizontal tangent must occur at the point of inflection.*

7. (5 points) Evaluate the following integral

$$\int \left(\frac{\sqrt{a^2 - x^2}}{x^4} + \frac{2x}{1 + 3x^2} + \sec^2 x \right) dx.$$

8. (5 points) Let a and b be nonzero real numbers and evaluate the integral

$$\int e^{ax} cos(bx)\, dx.$$

9. (5 points) For $x \neq 0$ define the function f by

$$f(x) = \int_0^x \frac{1}{1+t^2}\, dt + \int_0^{\frac{1}{x}} \frac{1}{1+t^2}\, dt.$$

Show that $f(x)$ is constant for $x \in (0, \infty)$ and $f(x)$ is also constant for $x \in (-\infty, 0)$. Finally, find the value of these two constants. *Hint: Recall that a function is constant if and only if its derivative is* 0 *everywhere in its domain of definition.*

10. (5 points) Find the value of $a > 0$ such that when the area bounded by the curve $y = \sqrt{x}e^{x^2} = f(x)$, the x-axis, and the line $x = a$ is rotated about the x-axis, a volume of 2π is generated.

11. (5 points) *(Bonus Question)* For all $a, b > 0$, show that

$$\lim_{x\to 0} \frac{(ab)^x - 1}{x} = \lim_{x\to 0} \frac{a^x - 1}{x} + \lim_{x\to 0} \frac{b^x - 1}{x}.$$

(*Scratch Work*)

Formulas You Might Need

Trigonometric Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v \qquad \sin(u-v) = \sin u \cos v - \cos u \sin v$$
$$\cos(u+v) = \cos u \cos v - \sin u \sin v \qquad \cos(u-v) = \cos u \cos v + \sin u \sin v$$
$$\sin(2u) = 2\sin u \cos u \qquad \sin^2 u = \frac{1-\cos(2u)}{2}$$
$$\cos^2 u = \frac{1+\cos(2u)}{2} \qquad \sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc x \qquad \tan^2 x + 1 = \sec^2 x$$

Differentiation Formulas

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\operatorname{arc\,cot} x = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\operatorname{arc\,sec} x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\operatorname{arc\,csc} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Substitutions

Integrals involving $\sqrt{a^2-x^2}$ we use the substitution $x = a\sin u$.

Integrals involving $\sqrt{a^2+x^2}$ we use the substitution $x = a\tan u$.

Integrals involving $\sqrt{x^2-a^2}$ we use the substitution $x = a\sec u$.