

Linear Independence

Math 1522
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If A is an $m \times n$ matrix, the following are equivalent (TFAE):

- (1) The columns of A are linearly independent (LI)
- (2) The equation $A\vec{x} = \vec{b}$ has at most one solution for any $\vec{b} \in \mathbb{R}^m$.
- (3) $\text{nullity}(A) = 0$
- (4) $\text{rank}(A) = n$
- (5) The columns of the reduced row echelon form (RREF) of A are all standard vectors of \mathbb{R}^m .
- (6) The only solution of $A\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$.
- (7) Each column of A is a pivot (no free variables).
- (8) If A is square ($m = n$) then A is invertible.
- (9) A is one-to-one.
- (10) The null-space of A consists of the just the $\mathbf{0}$ -vector.
- (11) If A is square ($m = n$) then $\det(A) \neq 0$.
- (12) If A is square ($m = n$) then 0 is not an eigenvalue of A .