MATH 2403 N1/N2/N3 **Name:** ____________________

Quiz #3 January 28, 2010

1. (5 points) Consider the following differential equation

$$\left(\frac{y}{x} + 6x\right) dx + (\ln x - 2)\, dy = 0.$$

a. Is the equation exact? *Note: You must show the condition for exactness is satisfied or not satisfied.*

Solution: Let $M(x, y) = \frac{y}{x} + 6x$ and $N(x, y) = \ln x - 2$. The equation is exact iff $M_y = N_x$. So, we compute $M_y = \frac{1}{x}$ and $N_x = \frac{1}{x}$ and discover that the equation is exact.

b. If the equation is exact, then solve the equation.

Solution: Now, we need to find a function $\psi(x, y)$ such that $\psi_x = M$ and $\psi_y = N$. So, we integrate N with respect to y to get

$$\begin{aligned} \psi(x, y) &= \int N(x, y) dy \\ &= \int (\ln x - 2) dy \\ &= y \ln x - 2y + h(x). \end{aligned}$$

Now, we find $h(x)$ by seeing that

$$\begin{aligned} M(x, y) &= \psi_x(x, y) \\ &= \frac{y}{x} + h'(x). \end{aligned}$$

Comparing the left hand and right hand sides of the previous equation we get $h'(x) = 6x$ which gives $h(x) = 3x^2$. Finally, the solution to the exact equation is

$$y \ln x - 2y + 3x^2 = c$$

where c is an arbitrary constant.

Please circle your section number: **N1 (Ruidong)** **N2 (Alex)** **N3 (Jackie)**