

Math 2605 Quiz 10

April 8, 2010

Name:

- (7 points) Let $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Find a reflection matrix M mapping \mathbf{x} to a multiple of \mathbf{e}_1
- (3 points) Compute the adjoint of $A = \begin{bmatrix} 1+i & i \\ 2-i & 3+i \end{bmatrix}$.

1. solution:

$$\|\vec{x}\| = \sqrt{1+4+4} = 3$$

Find M mapping \vec{x} to $3\mathbf{e}_1$.

$$\mathbf{u} = \vec{x} - \vec{e}_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\mathbf{u} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$M = I - 2\mathbf{u}\mathbf{u}^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

2. Solution:

$$\begin{bmatrix} 1-i & 2+i \\ -i & 3-i \end{bmatrix}$$