Math 2605-M Quiz 3 **Name:** SOLUTIONS

28 Jan 10

Consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$f(x, y) = \frac{sin(x)}{cos(y)}$

1. (5 points) Calculate $\frac{\partial f}{\partial x}(\pi, 0)$and $\frac{\partial f}{\partial y}(\pi, 0)$.
2. (5 points) Prove, by quoting an appropriate theorem if necessary, that f is continuous at the point $(0, 0)$.

(1) $f\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\sin(x)}{\cos(y)} = \sin(x)\sec(y)$

$\frac{\partial f}{\partial x} = \cos(x)\sec(y) \Rightarrow \frac{\partial f}{\partial x}\begin{bmatrix} \pi \\ 0 \end{bmatrix} = \cos(\pi)\sec(0) = -1$

$\frac{\partial f}{\partial y} = \sin(x)\sec(y)\tan(y) \Rightarrow \frac{\partial f}{\partial y}\begin{bmatrix} \pi \\ 0 \end{bmatrix} = 0 \cdot 1 \cdot 0 = 0$

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(2) See CAR p. 7 of Chapter 1, Theorem 1.

Let $g(x,y) = \sin(x)$ and $h(x,y) = \cos(y)$

Both g and h are continuous (in fact they are differentiable, everywhere), and h is not zero near $(0,0)$. So the Theorem implies that $f = g/h$ is continuous at $(0,0)$

Many of you cited the existence and continuity of the partial derivatives of f near $(0, 0)$.

For a discussion of why this is <u>not sufficient</u>, please see CAR, Chapter 1, section 2.3, beginning on page 18 and continuing through p. 19.