

Math 2605 Quiz 4

Feb. 4, 2010

Name:

1. (5 points)  $f(x, y) = 2x^2 + xy^3 + y^2$ ,  $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ . Define  $g(t) = f(\mathbf{x}_0 + t\mathbf{v})$ . Compute  $g'(1)$  using the gradient of  $f$ .

2. (5 points) Calculate the partial derivatives:

$$f(x, y, z) = z^{xy^2}$$

[hint: Take logarithm of  $f$ .]

$$\underline{1.} \quad g'(t) = \nabla f(\mathbf{x}_0 + t\mathbf{v}) \cdot \mathbf{v}$$

$$\Rightarrow g'(1) = \nabla f(3, 3) \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\nabla f(x, y) = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 4x + y^3 \\ 3xy^2 + 2y \end{bmatrix}$$

$$\Rightarrow \nabla f(3, 3) = \begin{bmatrix} 12 + 27 \\ 81 + 6 \end{bmatrix} = \begin{bmatrix} 39 \\ 87 \end{bmatrix}$$

$$\Rightarrow g'(1) = 39 \cdot 2 + 87 \cdot 1 = \boxed{165}$$

$$\underline{2.} \quad \ln f = xy^2 \ln z$$

$$\left. \begin{array}{l} \frac{f_x}{f} = y^2 \ln z \Rightarrow \underline{f_x} = z^{xy^2} y^2 \ln z \\ \frac{f_y}{f} = 2yx \ln z \Rightarrow \underline{f_y} = z^{xy^2} 2yx \ln z \\ \frac{f_z}{f} = \frac{xy^2}{z} \Rightarrow \underline{f_z} = z^{xy^2-1} xy^2 \end{array} \right\}$$