Math 2605-M Quiz 5 **Name:** SOLUTIONS

11 Feb 10

Consider the surface in $\mathbb{R}^3$ defined by

$$x + y + 6z + xy - x^2 - y^2 = 0$$

1. (5 points) Find an equation for the tangent plane to the surface at the point $\mathbf{p} = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}$.

2. (5 points) Find a point where the tangent plane is horizontal. What is the equation of the tangent plane there?

① Let $f(x,y,z) = x + y + 6z + xy - x^2 - y^2$

Then $\nabla f = \begin{bmatrix} 1 - 2x + y \\ 1 + x - 2y \\ 6 \end{bmatrix}$

$\nabla f(\vec{p}) = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$ ⟵ this is our normal vector $\vec{a}$.

the basepoint is $\vec{x}_0 = \vec{p}$

write $\vec{a} \cdot \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \vec{x}_0 \right) = 0 \Rightarrow \boxed{8x - 7y + 6z = -19}$

② the tangent plane is horizontal when its normal vector is vertical, that is, has form $\begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$.

The normal vector is just ∇f, so we want points where $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0$, so we set:

$$\boxed{\begin{aligned} 1 - 2x + y &= 0 \\ 1 + x - 2y &= 0 \end{aligned}}$$

and get $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

to write the point in $\mathbb{R}^3$ we plug $x = 1$ and $y = 1$ into $f(x, y, z) = 0$, and solve for z.

$$1 + 1 + 6z + 1 - 1 - 1 = 0 \Rightarrow \boxed{z = -1/6}$$

so at $\boxed{\vec{p} = \begin{bmatrix} 1 \\ 1 \\ -1/6 \end{bmatrix}}$ the gradient is $\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$

and the tangent vector, being parallel to the xy plane, has equation $\boxed{z = -1/6}$