

Math 2605 Quiz 6

Feb. 25, 2010

Name:

Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}.$$

- (5 points) Find the eigenvalues of  $A$ .
- (5 points) Find the eigenvectors of  $A$ . What is an orthogonal matrix  $U$  that diagonalizes  $A$ .

1.  $a=1$   $b=2$   $d=5$

$$\mu_1 = \frac{a+d}{2} + \sqrt{b^2 + \left(\frac{a-d}{2}\right)^2} = 3 + 2\sqrt{2}$$

$$\mu_2 = 3 - 2\sqrt{2}$$

2.  $\vec{v}_1$  satisfies

$$(A - \mu_1 I) \vec{v}_1 = 0 \Rightarrow \begin{pmatrix} -2-2\sqrt{2} & 2 \\ 2 & 2-2\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{v}_1 = \begin{pmatrix} \sqrt{2}-1 \\ 1 \end{pmatrix} \xrightarrow{\text{Normalization}} \frac{1}{\sqrt{4-2\sqrt{2}}} \begin{pmatrix} \sqrt{2}-1 \\ 1 \end{pmatrix}$$

$\vec{v}_2$  satisfies

$$(A - \mu_2 I) \vec{v}_2 = 0 \Rightarrow \begin{pmatrix} -2+2\sqrt{2} & 2 \\ 2 & 2+2\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{v}_2 = \begin{pmatrix} -1 \\ \sqrt{2}-1 \end{pmatrix} \xrightarrow{\text{Normalization}} \frac{1}{\sqrt{4-2\sqrt{2}}} \begin{pmatrix} -1 \\ \sqrt{2}-1 \end{pmatrix}$$

$$U = (\vec{v}_1 \ \vec{v}_2) = \frac{1}{\sqrt{4-2\sqrt{2}}} \begin{pmatrix} \sqrt{2}-1 & -1 \\ 1 & \sqrt{2}-1 \end{pmatrix}$$