Math 2605-M Quiz 7 **Name:** SOLUTIONS
4 March 10

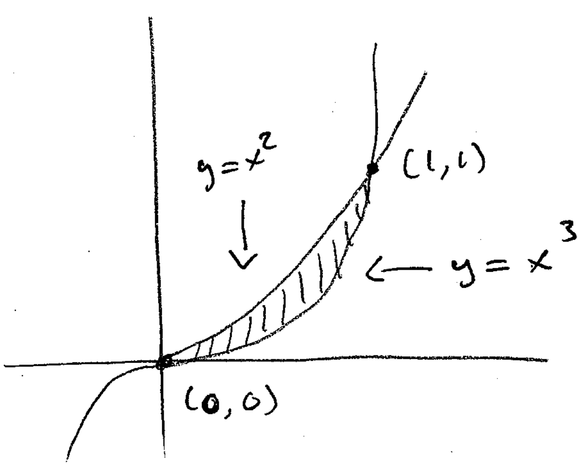
Let Ω be the region in the first quadrant of the xy plane bounded by the curves $y = x^2$ and $y = x^3$.

1. (7 points) Compute the following double integral over Ω.

$$\iint_{\Omega} (5x^2 + 2y)\, dx\, dy$$

2. (3 points) Let $f(x, y) = 5x^2 + 2y$, i.e. the function you just integrated. Compute the average value of f on Ω.

(1)



intersection points:

$$x^2 = x^3$$

$$x^2(x-1) = 0$$

$$\Rightarrow (0,0) \text{ and } (1,1)$$

So we have: $0 \le x \le 1$, $x^3 \le y \le x^2$

$$\int_0^1 \int_{x^3}^{x^2} (5x^2 + 2y)\, dy\, dx$$

$$= \int_0^1 \left(5x^2y + y^2 \Big|_{x^3}^{x^2} \right) dx$$

$$= \int_0^1 \left(5x^4 + x^4 - 5x^5 - x^6 \right) dx$$

$$= \frac{6}{5}x^5 - \frac{5}{6}x^6 - \frac{1}{7}x^7 \Big|_0^1 = \frac{6}{5} - \frac{5}{6} - \frac{1}{7}$$

$$= \boxed{\frac{47}{210}}$$

(2) Area of $\Omega = \int_0^1 \int_{x^3}^{x^2} dy\, dx$

$$= \int_0^1 (x^2 - x^3)\, dx = \frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1 = \frac{1}{12}$$

$\Rightarrow$ Avg. value of $f = \dfrac{\iint_\Omega f \, dx\, dy}{\text{Area } \Omega}$

$$= \frac{47}{210} \Big/ \frac{1}{12} = \boxed{\frac{94}{35}}$$