

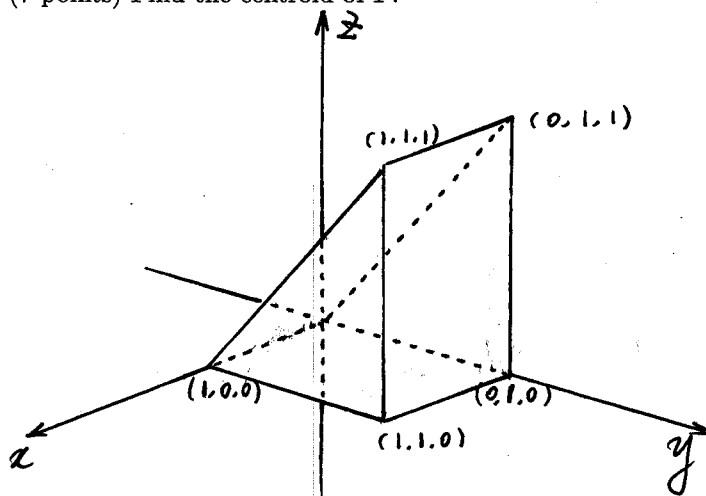
Math 2605 Quiz 8

Mar. 11, 2010

Name:

Let T be the solid bounded above by the plane $z = y$, below by the xy -plane, on the sides by the planes $x = 0, x = 1, y = 1$.

1. (3 points) Find the volume of T
2. (7 points) Find the centroid of T .



$$1) \quad T = (1 \cdot 1) \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$\begin{aligned} 2) \quad \bar{x}V &= \int_0^1 \int_0^1 \int_0^y x \, dz \, dy \, dx \\ &= \int_0^1 \int_0^1 xz \Big|_0^y \, dy \, dx \\ &= \int_0^1 \int_0^1 xy \, dy \, dx \end{aligned}$$

$$= \int_0^1 \frac{1}{2}xy^2 \Big|_0^1 \, dx = \int_0^1 \frac{1}{2}x \, dx = \frac{1}{4}x^2 \Big|_0^1 = \frac{1}{4}$$

$$\Rightarrow \bar{x} = \frac{1}{4} / \left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\begin{aligned}
\bar{y}V &= \int_0^1 \int_0^1 \int_0^y y \, dz \, dy \, dx \\
&= \int_0^1 \int_0^1 zy \Big|_0^y \, dy \, dx \\
&= \int_0^1 \int_0^1 y^2 \, dy \, dx \\
&= \int_0^1 \frac{1}{3} y^3 \Big|_0^1 \, dx \\
&= \int_0^1 \frac{1}{3} \, dx = \frac{1}{3}
\end{aligned}$$

$$\bar{y} = \frac{1/3}{(1/2)} = \frac{2}{3}$$

$$\begin{aligned}
\bar{z}V &= \int_0^1 \int_0^1 \int_0^y z \, dz \, dy \, dx \\
&= \int_0^1 \int_0^1 \frac{1}{2} z^2 \Big|_0^y \, dy \, dx \\
&= \int_0^1 \int_0^1 \frac{1}{6} y^2 \, dy \, dx \\
&= \int_0^1 \frac{1}{6} y^3 \Big|_0^1 \, dx \\
&= \int_0^1 \frac{1}{6} \, dx = \frac{1}{6}
\end{aligned}$$

$$\bar{z} = \frac{1/6}{(1/2)} = \frac{1}{3}$$

So the centroid is $(\frac{1}{2}, \frac{2}{3}, \frac{1}{3})$.