Instructions: 1. Closed book, calculators may be used.
   2. Show your work and explain your answers and reasoning.
   3. Express your answers in simplified form.

1. (25) Compute

   a. \( \lim_{x \to 0} \frac{\tan(2x)}{3x} \)
   b. \( \frac{d}{dx} \left[ \left( \frac{x-1}{x+1} \right)^2 \right] \)
   c. \( f'(x) \) and \( f''(x) \) if \( f(x) = x^2 e^{-x} \).
   d. \( \frac{d}{dx} x \arcsin(x) \)
   e. \( \frac{d}{dx} y \) at \( (1, -2) \), given that \( 3x^2 + xy + y^2 = 5 \).

2. (25) Evaluate

   a. \( \int x \left( x^2 + 4 \right)^7 \, dx \)
   b. \( \int_0^\infty \sec^2 x (1 + \tan x)^2 \, dx \)
   c. \( \int x e^{-4x} \, dx \)
   d. \( \int_0^2 \frac{x^2}{x^2 + 4} \, dx \)
   e. \( \int \frac{dx}{(x+1)(x-1)} \)

3. (25) Sand is being dropped onto the top of a conical pile at the rate of 3 cubic meters per minute. Suppose the height of the pile is always equal to its diameter. How fast is the height increasing when the pile is 6 meters high?

4. (25) An open topped cylindrical pot is to have volume 125 cubic inches. What dimensions minimize the total amount of material used in making the pot? (Please do not consider the thickness of the pot or any waste in the manufacturing process.)

5. (25) The region \( \Omega \) shown to the right is bounded by the graphs of \( y = \cos(x) \), \( y = \sin(x) \), and the positive \( y \)-axis.

   a. Find the area of \( \Omega \).
   b. Find the volume of the solid obtained by revolving \( \Omega \) about the \( y \)-axis.
6. (25) The reservoir shown below is formed by revolving the part of the hyperbola 
\[ x^2 - y^2 = 100, \quad x \geq 0, \quad 0 \leq y \leq 20 \] 
around the y-axis (measurements in feet). It is filled 
to a depth of 4 feet with water with a density of 62.5 pounds per cubic foot. How 
much work is required to empty the tank by pumping the water to the top of the tank 
and spilling it?

7. (25) Express the following quantities in the form \( a + bi \). Your answers should 
contain no decimals and no trigonometric functions.

a. \((2 + 3i)(3 - i)\)  
   b. \(\frac{5 + 12i}{1 + 3i}\)  

   c. All complex cube roots of \(8i\). (That is, all complex solutions of \(z^3 = 8i\).)

Answers.

1. a. \(\frac{2}{3}\)  
   b. \(\frac{4(x - 1)}{(x + 1)^2}\)  
   c. \(f'(x) = 2xe^{-x} - x^2 e^{-x}, \quad f''(x) = 2e^{-x} - 4xe^{-x} + x^2 e^{-x}\)  
   d. \(\sin^{-1}(x) + \frac{x}{\sqrt{1 - x^2}}\)  
   e. \(\frac{4}{3}\)

2. a. \(\frac{1}{16}(x^2 + 4)^8 + C\)  
   b. \(\frac{7}{3}\)  
   c. \(-\frac{x}{4}e^{-4x} - \frac{1}{16}e^{-4x} + C\)  
   d. \(2 - \frac{\pi}{2}\)  
   e. \(-\frac{1}{2}\ln|x + 1| + \frac{1}{2}\ln|x - 1| + C\)
3. \( \frac{1}{3\pi} \)

4. \( h = r = \frac{5}{\pi^2} \)

5. a. \( \sqrt{2} - 1 \)      
     b. \( 2\pi \left( \frac{\pi \sqrt{2}}{4} - 1 \right) \)

6. \( (62.5\pi) \left( \frac{22,688}{3} \right) \)

7. a. \( 9 + 7i \)      
     b. \( \frac{41}{10} + \frac{-3}{10}i \)      
     c. \( \sqrt{3} + i, -\sqrt{3} + i, -2i \)