Instructions: 1. Closed book, calculators may be used.
2. Show your work and explain your answers and reasoning.
3. Express your answers in simplified form.

1. (25) Compute
   a. \( \frac{d}{dx} \left( \ln(x^2 + 2) + \sin^{-1}(3x) \right) \)
   b. \( \int \frac{e^x}{1 - e^x} \, dx \)
   c. \( \int \frac{\ln x}{x} \, dx \)
   d. \( \frac{d}{dx} \left( \frac{e^x + e^{-x}}{2} \right) \)
   e. \( \frac{d}{dx} \left( \frac{e^x - e^{-x}}{2} \right) \)

2. (25) Find
   a. \( \int x \ln(x) \, dx \)
   b. \( \int \sin(x)\sin(2x) \, dx \)
   c. \( \int \sin^2(x)\cos^3(x) \, dx \)
   d. \( \int_0^5 x^2 \sqrt{25 - x^2} \, dx \)
   e. \( \int_0^\pi x \sin(x) \, dx \)

3. (25) Cobalt-60, which is used extensively in medicine, has a half-life of 5.3 years.
   a. What percentage of a given amount of cobalt-60 will remain after 8 years?
   b. If a sample has 100 grams of cobalt-60 now, how much was there 3 years ago?
      (Remember that the rate of decay of a radioactive substance is proportional to the amount of the substance present.)
4. (25) The region bounded by the graph of $f(x) = \frac{1}{1 + x^2}$, the $x$-axis, the $y$-axis, and the line $x = 1$, is shown at the right.

a. Compute the volume of the solid obtained by revolving this region about the $y$-axis.

b. Compute the volume of the solid obtained by revolving this region about the $x$-axis.

Answers

1. a. \( \frac{2x}{x^2 + 2} + \frac{3}{\sqrt{1 - 9x^2}} \)  
   b. \(-\ln|1 - e^x| + C\)
   c. \(\ln|\ln x| + C\)  
   d. \(\frac{e^x - e^{-x}}{2}\)  
   e. \(\frac{e^x + e^{-x}}{2}\)

2. a. \(\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C\)  
   b. \(\frac{1}{2}\sin(x) - \frac{1}{3}\sin(3x) + C\)
   c. \(\frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C\)  
   d. \(\frac{625\pi}{16}\)  
   e. \(\pi\)

3. a. 35%  
   b. 148 grams

4. a. \(\pi \ln(2)\)  
   b. \(\pi \left(\frac{\pi}{8} + \frac{1}{4}\right)\)