Instructions: 1. Closed book, calculators may be used.
   2. Show your work and explain your answers and reasoning.
   3. Express your answers in simplified form.

1. (25) Compute
   a. \( \int \left( \sec(x) \tan(x) + \sqrt[3]{x} \right) dx \)
   b. \( \int_{0}^{1} \frac{r}{(1 + r^2)^4} dr \)
   c. \( \int x \sqrt{x + 1} \, dx \)
   d. \( \int_{0}^{\pi/2} (3 + \cos(2x)) \, dx \)

2. (25) Sketch the region bounded by the three curves
   
   \[ y = x, \]
   \[ y = 12 - x^2 \quad (x \leq 0), \]
   \[ y = 12 - x \quad (x > 0) \]

   and find its area.

3. (25) The region \( \Omega \) shown at the right is bounded by the y-axis and the curves \( y = 10 - x \) and \( y = x^3 \). Find the volume of the solid generated by revolving \( \Omega \) about the y-axis. (Note: the curves \( y = 10 - x \) and \( y = x^3 \) intersect when \( x = 2 \).)

4. (25) A vertical cylindrical tank of radius 3 feet and height 5 feet is full of water. Find the work done in pumping out the water:
   a. to an outlet at the top of the tank;
   b. to a level 4 feet above the top of the tank.

   (Assume that the water weighs 62.5 pounds per cubic foot.)
Answers

1. a. \( \sec(x) + \frac{4}{5}x^5 + C \)  
   b. \( \frac{7}{48} \)
   c. \( \frac{2}{5}(x + 1)^5 - \frac{2}{3}(x + 1)^3 + C \)  
   d. \( 3\pi \)

2. \( \frac{212}{3} \)

3. \( \frac{328\pi}{15} \)

4. a. \( \frac{225\pi\sigma}{2} \)  
   b. \( \frac{585\pi\sigma}{2} \)  
   where \( \sigma = 62.5 \)