Instructions: 1. Closed book, calculators may be used. 
2. Show your work and explain your answers and reasoning.

1. (25) a. Evaluate the double integral 
\[ \iint_{\Omega} ye^x \, dxdy \quad \Omega : \quad 0 \leq y \leq 1, \quad 0 \leq x \leq y^2. \]

b. Evaluate 
\[ \int_{y=0}^{1} \int_{x=1}^{2y} (x + 2z) \, dz \, dx \, dy. \]

2. (25) A solid right circular cylinder T is defined by \( x^2 + y^2 \leq 4, \ 0 \leq z \leq 5 \), and has mass density given by \( \lambda(x,y,z) = z \). Compute the moment of inertia of T about the z-axis.

3. (25) Find the mass of a ball of radius R given that the density varies directly with the distance from the boundary.

4. (25) a. \( h(x,y) = (2xy + 1)i + x^2j \). Find a function \( f(x,y) \) with 
\[ \nabla f(x,y) = h(x,y) \] and use it to evaluate \( \int_C h(r) \cdot dr \) where \( C \) is a path from \((1,0)\) to \((3,4)\).

b. Use Green's Theorem to evaluate 
\[ \oint_C (3y \, dx + 5x \, dy) \quad C : \quad x^2 + y^2 = 1. \]

Answers to Hour Test 3

1. a. \( \frac{e}{2} - 1 \). B. 2/3

2. 100\pi

3. Using spherical coordinates, the density is \( \lambda = k(R - \rho) \), and \( M = \frac{\pi k R^4}{3} \).

4. a. \( f(x,y) = x^2y + x \), \( \int_C h(r) \cdot dr = f(3,4) - f(1,0) = 38 \)

b. 2\pi