Instructions: 1. Closed book, calculators may be used.
   2. Show your work and explain your answers and reasoning.

1. (25) A small bug starts walking at time $t = 0$, with position given by
   \[ \mathbf{r}(t) = t\mathbf{i} + \frac{t^2}{\sqrt{2}}\mathbf{j} + \frac{t^3}{3}\mathbf{k}. \] (distances in feet)
   
a. How far has the bug traveled when $t = 1$?
   
b. At what time $t$ has the bug traveled 12 feet?

2. (25) The function $f(x, y) = \frac{3y}{2} - \frac{y^3}{2} - x^2 y + 4$ has four stationary points.
   Find them, and classify each one as a local minimum, local maximum, or saddle point.

3. (25) A rectangular box with no lid is to have volume 4000 cubic centimeters. What
   should the dimensions be in order to minimize its total surface area?

4. (25) Find the volume under the paraboloid $z = 17 - x^2 - y^2$ within the cylinder
   $x^2 + y^2 \leq 2$.

5. (25) Calculate the moment of inertia of the hemisphere
   \[ H = \{(x,y,z) : x^2 + y^2 + z^2 \leq 4, z \geq 0\}, \]
   about the $z$-axis, given that the density is proportional to the distance from the $x$-$y$
   plane.

6. (25) Let $S$ be the surface (lateral, top, and bottom) of the cylinder
   \[ \{(x,y,z) : x^2 + y^2 \leq 5, \quad \mathbf{z} \leq 3\}. \]
   Use the Divergence Theorem to calculate the flux of the vector field
   \[ \mathbf{F}(x,y,z) = yi + xj + z^2k \] through $S$. 
Answers to Final Exam

1. a. $\frac{4}{3}$. b. 3.

2. $(0,1)$ is a local maximum, $(0,-1)$ is a local minimum, $\left(\sqrt{\frac{3}{2}}, 0\right)$ and $\left(-\sqrt{\frac{3}{2}}, 0\right)$ are saddle points.

3. 20 by 20 by 10.

4. $32\pi$.

5. $16k\pi/3$.

6. $45\pi$. 