1. (25) Find an equation for the tangent plane to

\[ z = x^2 + xy + y^2 - 6x + 2 \]

at (4, 3, 15).

2. (25) The volume of a right circular cone with base radius \( r \) and height \( h \) is

\[ V = \frac{\pi}{3} r^2 h \].

Suppose the radius is measured to be 6 inches ± .2 inches, and the height is measured to be 12 inches ± .3 inches. Use differentials to estimate the maximum possible error in the calculated volume.

3. (25) A rectangular box is to have volume 48 cubic feet, and is made of three different grades of material. The material for the front and back costs $1 per square foot, the material for the top and bottom costs $2 per square foot, and the material for the two ends costs $3 per square foot. What are the dimensions of the box of minimal cost?

4. (25) Let \( g(x, y) = y - x^2 \) and let \( f(x, y) = x^2 + (y-1)^2 \).

a. On the axes provided, sketch the parabola \( g(x, y) = 0 \) and sketch \textbf{two} level curves of \( f(x, y) \).

b. Use the method of Lagrange multipliers to find the minimum value of \( f(x, y) \) subject to \( g(x, y) = 0 \).

Answers to Test II

1. \( z = 15 + 5(x - 4) + 10(y - 3) \)

2. \( 13.2\pi \)

3. The dimensions are 2' by 4' by 6'

4. The minimum value is \( 3/4 \).