Instructions: 1. This is a closed book exam. You may use a calculator. 
2. Show your work and explain your answers and reasoning.

1. (25) Find the least distance between the lines 
   \[ x(t) = (t+1)i + t j + (-t-3)k \] and 
   \[ y(s) = (-2s+3)i + (s+1)j + (-s-2)k. \] 
   (Suggestion: Begin by finding the minimum value of \( f(s,t) = \) squared distance from \( x(t) \) to \( y(s) \).)

2. (25) Use the method of Lagrange multipliers to find the maximum value of \( xy \) subject to the condition that \( x + 2y = 400 \).

3. (25) Evaluate \( \int \int_R e^{x^2+y^2} dA \) where \( R \) is the half disk 
   \[ \{(x,y): 0 \leq y \leq \sqrt{1-x^2}, -1 \leq x \leq 1 \}. \]

4. (25) This problem deals with the integral \( I \), which is expressed below in cylindrical coordinates.
   \[ I = \int_{\theta=0}^{2\pi} \int_{r=0}^{2\sqrt{4-r^2}} \int_{z=0}^{2-r} r^2 dz dr d\theta. \]
   a. Sketch the domain of integration \( D \), and describe \( D \) in a complete sentence containing no more than 25 words.
   b. Write \( I \) as an iterated integral in spherical coordinates.
   c. Write \( I \) as an iterated integral in Cartesian coordinates.
   d. Calculate \( I \) using any one of these three expressions.

5. (25) The hemispherical tank shown has radius 5 meters and is filled with liquid to within 3 meters of the top. What is the volume of this liquid?
6. (25) Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = (y+1)\mathbf{i}$, and let $C$ be the curve composed of a semicircle and four line segments sketched below.

a. Indicate the vector field $\mathbf{F}$ by drawing at least 8 arrows on the figure at the left. Please include arrows in all four quadrants.

b. Calculate $\int_C \mathbf{F} \cdot d\mathbf{x}$.

7. (25) By direct calculation compute

$$\iint_S (z\mathbf{i} + x^2\mathbf{k}) \cdot d\mathbf{σ}$$

where $S$ is the portion of the surface $z = x^2 + y^2$ which lies above the square $\{(x,y): \ -1 \leq x \leq 1, -1 \leq y \leq 1\}$, oriented so that the unit normal $\mathbf{n}$ points upward.

8. (25) Use the divergence theorem to calculate the flux of $\mathbf{F}(x,y,z) = x\mathbf{i} + (3y+z)\mathbf{j}$ out of the surface of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + 3z = 12$. 