Instructions:
1. This is a closed book exam. You may use a calculator and notes on one 8.5 by 11 inch sheet of paper.
2. Show your work and explain your answers and reasoning.

1. (25) Evaluate each of the following limits.
   a. \( \lim_{x \to 0} \frac{\sec(x) - 1}{x^2} \)
   b. \( \lim_{x \to 0} (x \ln(x)) \)
   c. \( \lim_{x \to \infty} \frac{x^2 + 2x}{e^{3x}} \)

2. (25) Determine which series converge and which diverge.
   a. \( \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} \)
   b. \( \sum_{n=2}^{\infty} \frac{1}{n \ln(n)} \)
   c. \( \sum_{n=2}^{\infty} \frac{n^2 + 3n}{4n^3 - n + 2} \)
   d. \( \sum_{n=2}^{\infty} ne^{-n} \)

3. (25) a. By substituting \( t^2 \) for \( t \) in the Taylor series, centered at 0, for \( f(t) = \sin(t) \), obtain the Taylor series, centered at 0, for \( \sin(t^2) \).
   
   b. Use your result from part a to calculate the Taylor series, centered at 0, for the function \( G(t) \) which satisfies \( \frac{dG}{dt} = \sin(t^2) \) and \( G(0) = 0 \).
4. (25) Let P, Q, and R be the points (3, 6, 10), (0, 4, 8), and (1, 5, 7), respectively.
   
   a. Is the triangle PQR a right triangle? If so, at which vertex is the right angle?
   
   b. What is the area of triangle PQR?

5. (25) Let \( u_1 = (1, 1, 2) \), \( u_2 = (2, 0, 2) \), \( u_3 = (-3, 2, 5) \).
   
   a. Find constants \( a_1 \), \( a_2 \), \( a_3 \), so that \( a_1 u_1 + a_2 u_2 + a_3 u_3 = (13, 0, 7) \).
   
   b. Show that every \( w \) in \( \mathbb{R}^3 \) is a linear combination of \( u_1 \), \( u_2 \), \( u_3 \).

6. (25) Find the standard matrix for the stated composition of linear operators on \( \mathbb{R}^3 \).
   
   a. Reflection about the \( x-z \) plane, followed by counter-clockwise rotation about the \( y \)-axis by \( \frac{\pi}{4} \) radians, followed by orthogonal projection onto the \( y-z \) plane.
   
   b. Counter-clockwise rotation about the \( y \)-axis by \( \frac{\pi}{4} \) radians, followed by orthogonal projection onto the \( y-z \) plane, followed by counter-clockwise rotation about the \( y \)-axis by \( \frac{\pi}{4} \) radians, followed by reflection about the \( x-z \) plane.

7. (25) The characteristic polynomial of the matrix

\[
A = \begin{pmatrix} 2 & -4 & -8 \\ -2 & 3 & 7 \\ 2 & -2 & -6 \end{pmatrix}
\]

is \( p(\lambda) = |\lambda I - A| = \lambda^3 + \lambda^2 - 2\lambda \).
   
   a. Determine the eigenvalues and eigenvectors of \( A \).
   
   b. Find an invertible matrix \( P \) and a diagonal matrix \( D \) such that \( D = P^{-1}AP \).