Instructions: 1. This is an open book exam. You may use the texts by Grossman, Anton, and your class notes. You may use a calculator.
2. Show your work and explain your answers and reasoning.

1. (25) Determine whether the following series converge or diverge. Be sure to explain your reasoning.
   a. \[ \sum_{n=1}^{\infty} \frac{n + 1}{7^n} \]
   b. \[ \sum_{n=1}^{\infty} \frac{7^n}{n^2 + 2} \]
   c. \[ \sum_{k=3}^{\infty} \frac{(-1)^k}{\ln k} \]

2. (25) For each of the following power series, determine the radius of convergence and the interval of convergence (i.e., the interval in which the series converges.) Don't forget to check the endpoints of the interval of convergence.
   a. \[ \sum_{n=1}^{\infty} \frac{1}{n} (x-1)^n \]
   b. \[ \sum_{n=1}^{\infty} n^n x^n \]
   c. \[ \sum_{n=1}^{\infty} \frac{n^{n+1}}{n!} (x-3)^n \]

3. (25) Determine the Taylor Series for the following function, centered at the specified point \( x_0 \). In each case, what is the value of the coefficient \( a_6 \) (the coefficient of \( x^6 \)) .
   a. \( f(x) = \frac{x}{1+x^2} \), \( x_0 = 0 \).
b. \( g(x) = x^{\frac{1}{2}}, \ x_0 = 1. \)

4. (25) Determine the standard matrix for the following linear transformations.
   a. Rotation of \( \mathbb{R}^2 \) clockwise by \( \pi/3 \) radians followed by reflection across the y-axis.
   b. Clockwise (as viewed from the positive y-axis) rotation of \( \mathbb{R}^3 \) by \( \pi/2 \) radians followed by counter-clockwise (as viewed from the positive x-axis) rotation about the x-axis by \( \pi/2 \) radians.

5. (25) a. Explain why the vectors \( \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \) and \( \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \) span the plane with equation \( x + y - 2z = 0. \)
   b. Find the matrix for the orthogonal projection onto the plane of part a.

6. (25) What is the value of the variable \( x_2 \) in the least squares solution of the system of equations

\[
\begin{align*}
2x_1 - 2x_2 &= 2 \\
x_1 + x_2 &= -1 \\
3x_1 + x_2 &= 2.
\end{align*}
\]

7. (25) Let \( A \) be the 3 by 3 matrix \( A = \begin{pmatrix} 2 & 0 & -1 \\ 8 & -2 & -5 \\ 0 & 0 & 1 \end{pmatrix} \). Calculate the eigenvalues and eigenvectors of the matrix \( A \). For what matrix \( P \) is \( P^{-1}AP \) a diagonal matrix?

8. (25) Let \( A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix} \) and let \( P = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \).

   a. Verify that \( P^{-1}AP \) is a diagonal matrix
   b. What are the eigenvalues of \( A \)?
   c. Use the results above to calculate \( A^{1993} \).