1. a. Let $A$ and $B$ be non-empty sets in $\mathbb{R}$. Define $S = \{a+b: a \in A, b \in B\}$. Show that

$$\sup S = \sup A + \sup B$$

b. Show that $\sqrt{2}$ is an irrational number.

2. a. State the definition of “$K$ is a compact set in $\mathbb{R}^n$.”

b. Use the definition of compact set to show that

$$K = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$$

is compact in $\mathbb{R}$.

c. Determine the set of cluster points and the boundary set of $K$.

3. a. Let $A$ be a nonempty set in $\mathbb{R}^n$. Show that if $F \supseteq A$ and $F$ is closed, then $F \supseteq (A \cup A')$, where $A'$ denotes the set of cluster points of $A$.

b. Show that the set of rational numbers $\mathbb{Q}$ is neither open nor closed in $\mathbb{R}$.

4. True or false. Give a reason if you think it is true, and give a counterexample if you think it is false.

a. Every countable infinite set has empty interior.

b. Every countable set in $\mathbb{R}$ is closed in $\mathbb{R}$.

c. The union of two connected sets is connected.

d. The boundary of every closed connected set in $\mathbb{R}^2$ is connected.

e. Every uncountable set in $\mathbb{R}$ must have at least one cluster point.