1. Determine all point(s) at which \( f(z) = x^3 + y^2 i \) is differentiable, and calculate \( \frac{df}{dz} \) at the point(s) where it exists.

Answer: The derivative exists on the parabola \( y = \frac{3x^2}{2} \), and equals \( 3x^2 \).

2. Calculate a linear fractional transformation which maps \( z_1 = 0, z_2 = 1, z_3 = \infty \) to \( w_1 = -i, w_2 = 1, w_3 = i \), respectively. In addition

   a. Determine into what region this linear fractional transformation maps the upper half plane.
   
   b. Determine into what curve the line \( \text{Im}(z) = 1 \) is mapped by this linear fractional transformation.
   
   c. Determine into what curve the line \( \text{Im}(z) = -1 \) is mapped by this linear fractional transformation.

Answers: \( w = \frac{iz + 1}{z + i} \)

   a. unit disk.
   
   b. circle, center \( \frac{i}{2} \), radius \( \frac{1}{2} \)
   
   c. line \( \text{Im}(z) = 1 \).

3. Let \( g(z) \) be analytic inside and on a simple closed contour \( C \). Show that for all \( z_0 \) except those on \( C \),

\[
\int_C \frac{g(z)dz}{(z - z_0)^2} = \int_C \frac{g'(z)dz}{z - z_0}.
\]

What (common) value do these integrals have?

Answer: For \( z_0 \) inside \( C \) the value is \( 2\pi ig'(z_0) \). For \( z_0 \) outside, it is 0.

4. Let \( f(z) = \frac{e^z - 1}{z^4 (z^2 + 4)} \)
a. Calculate the residue of $f$ at 0 by calculating a few terms in the appropriate Laurent expansion for $f(z)$.

b. Calculate the residue of $f$ at $2i$.

c. What is the radius of convergence of the Taylor series for $f(z)$ centered at $2 + 3i$?

Answers:

a. The residue is $-1/48$.

b. $\frac{e^{2i} - 1}{64i}$

c. $\sqrt{5}$

5. Use the Residue Theorem to show that $\int_{0}^{\infty} \frac{x^2}{x^4 + 1} \, dx = \frac{\pi}{2\sqrt{2}}$. 