1. Compute all fourth roots of $-16i$, expressing your answers in the form $a + bi$. Sketch them on the complex plane, and indicate which is the principal root.

Answer: $z = 2\left(\cos\left(-\frac{\pi}{8} + \frac{k\pi}{2}\right) + i\sin\left(-\frac{\pi}{8} + \frac{k\pi}{2}\right)\right)$ The principal root is obtained by using the principal argument for $-16i$. The principal argument is $-\pi/2$, so the principal root occurs for $k = 0$ above.

2. Calculate all values of $\left(\sqrt{2}(1+i)\right)^i$. Sketch any three of them on the complex plane.

Answer: $e^{-\frac{i\pi}{4} + 2\pi k}(\cos(\ln(2)) + isin(\ln(2))), \quad k = 0, \pm1, \pm2,\ldots$

3. With $f(z) = x^3 + iy^3$, determine all points $z = x + iy$ at which $f$ is differentiable. Calculate $\frac{df}{dz}$ at those points.

Answer: $u = x^3, \quad v = y^3$. These have continuous partials at all points, and the Cauchy-Riemann equations are satisfied if and only if $y = \pm x$. At these points $\frac{df}{dz} = u_x + iv_x = 3x^2$.

4. Find a linear fractional transformation $w = \frac{az + b}{cz + d}$ which maps $z_1 = 0, \quad z_2 = i, \quad z_3 = \infty$ onto $w_1 = -1, \quad w_2 = -i, \quad w_3 = 1$, respectively. Show that the upper half-plane is mapped to the disk of radius 1 centered at 0, and sketch the image of the strip $\{z : 0 < \text{Im}(z) < 1\}$.

Answer: Using the cross-ratio, we see that $w = \frac{z - i}{z + i}$. Since $z_1, \quad z_2, \quad z_3$ determine the real axis and $w_1, \quad w_2, \quad w_3$ determine the unit circle, the real axis is mapped to the unit circle. Since $w(i) = 0$, the upper half plane is mapped to the interior of the unit disk. The line $\text{Im} \ z = 1$ must be mapped to a circle lying inside the unit disk and containing $w(i) = 0$ and $w(\infty) = 1$. This is the circe with center 1/2 and radius 1/2.