Instructions:

1. The test consists of 3 problems. Please make sure you have the entire test before you begin your work.
2. Notes are not permitted, do your work alone, the honor code applies.
3. You must show all your work in the space provided (you may use the backs of pages if necessary). Unsupported answers may receive no credit.

[5pts] 1. Let $(X_k)_{k=1}^{\infty}$ be a family of Bernoulli i.i.d. random variables with parameter 1/2: $\mathsf{P}\{X_k = 1\} = \mathsf{P}\{X_k = 0\} = 1/2$. Let (S_n) be the associated sequence of cumulative sums:

$$S_n = \begin{cases} 0, & n = 0, \\ X_1 + \ldots + X_n, & n \geq 1. \end{cases}$$

(a) Let $\tau_1 = \min\{n : \ S_n = 1\}$. Find the generating function of τ_1.

(b) Is it true that $\mathsf{P}\{\tau_1 < \infty\} = 1$? If yes, is $\mathsf{E}\tau_1$ finite?

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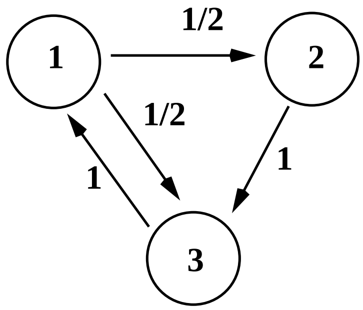

[5pts] 2. For the transition graph above, (i) find the transition matrix; (ii) find classes of communicating states and inessential states; (iii) for each class find its period; (iv) find all stationary distributions; (v) for each initial state consider the Markov chain with given transition probabilities and originatig at that state; does the distribution of this Markov chain at time n have a limit as $n \to \infty$? do the limits coincide for different initial states? If your answer to at least one of these questions is “no”, explain why this does not contradict the Perron–Frobenius theorem.

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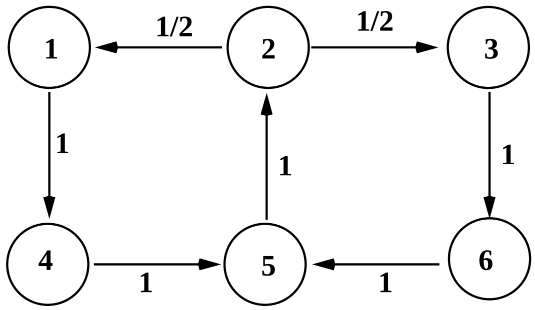

[5pts] 3. For the transition graph above, (i) find the transition matrix; (ii) find classes of communicating states and inessential states; (iii) for each class find its period; (iv) find all stationary distributions; (v) for each initial state consider the Markov chain with given transition probabilities and originatig at that state; does the distribution of this Markov chain at time n have a limit as $n \to \infty$? do the limits coincide for different initial states? If your answer to at least one of these questions is "no", explain why this does not contradict the Perron–Frobenius theorem.

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