subclass[RS[x], y]

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summary

A rewrite rule is derived which has as an application a reformulation of the \texttt{acl} version of the axiom of choice. The rewrite rule itself is generally valid, and does not depend on whether the axiom of choice does or does not hold.

derivation

Lemma 1. (Temporary rewrite rule using \texttt{thinarpt} wrapper.)

\begin{verbatim}
In[2]:= equal[V, image[inverse[
    IMAGE[composite[id[thinarpt[x]], inverse[FIRST]]], y]]] // AssertTest
\end{verbatim}

\begin{verbatim}
Out[2]= equal[V,
    image[inverse[IMAGE[composite[id[thinarpt[x]], inverse[FIRST]]]], y]] := subclass[RS[x], y]
\end{verbatim}

A better result will be derived shortly, eliminating the need for the \texttt{thinarpt} wrapper.

\begin{verbatim}
In[3]:= equal[V, image[inverse[IMAGE[composite[id[thinarpt[x]], inverse[FIRST]]]],
    y_]] := subclass[RS[x], y]
\end{verbatim}

Removing the \texttt{thinarpt} wrapper yields Lemma 2:
Lemma 3: The thin-ness hypothesis in Lemma 2 is not needed.

Using Lemma 3 to simplify Lemma 2 yields Lemma 4:

Lemma 5 is an implication in the opposite direction:

Main Theorem. Combining all these lemmas yields a logical equivalence.
This is made into a new rewrite rule.

\[ In[13]:= \text{equal}[V, \text{image}[	ext{inverse}[	ext{IMAGE}[	ext{composite}[\text{id}[x], \text{inverse}[\text{FIRST}]]], y])] := \text{and}[	ext{equal}[V, \text{domain}[	ext{VERTSECT}[x]], \text{subclass}[\text{RS}[x], y]]] \]

connection with the axiom of choice

When \( x \) is replaced with \( \text{inverse}[E] \) and \( y \) with \( \text{SELECT} \), one obtains a restatement of the \texttt{ac1} set-form of the axiom of choice.

\[ In[14]:= \text{equal}[V, \text{image}[	ext{inverse}[	ext{IMAGE}[	ext{composite}[\text{id}[\text{inverse}[E]], \text{inverse}[\text{FIRST}]]], \text{SELECT}]]] \]

\[ Out[14]= \text{axch} \]

The effect of the new rewrite rule is to transform this statement first to \( \text{subclass}[-\text{RS}[\text{inverse}[E]], \text{SELECT}] \), and then to \texttt{axch}. 