Quaife's corollary (LD3cor)

Johan G. F. Belinfante and Ming Li
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summary

The equality rewrite rules for natsub and monus in the GOEDEL program sometimes interfere with equality substitution. Such conflicts can be resolved by adding additional rewrite rules. A case in point is Quaife's corollary (LD3cor), which is related to Quaife's clause (LD4g) by a simple equality substitution. Although the GOEDEL program already has a rewrite rule analogous to (LD4g), this does not suffice for it to recognize the truth of corollary (LD3cor). The reason is that the GOEDEL program first rewrites the equation in (LD3cor), thereby blocking application of the equality substitution rule. To remedy this, an additional rewrite rule is derived in this notebook.

derivation

The GOEDEL program contains the following rewrite rule analogous to Quaife's clause (LD4g).

The GOEDEL also has the following rewrite rule for equations involving monus.

The following lemma helps clean up the result obtained when the above facts are combined.
Theorem. Analog of Quaife's (LD3cor). To avoid five numberhood literals, all variables are wrapped with nat.

\begin{verbatim}
In[8]:= SubstTest[implies,
    and[equal[t, w], member[t, ld[nat[x], nat[y]]],
        member[w, ld[nat[x], nat[y]]],
        {t -> monus[natmul[nat[u], nat[x]], natmul[nat[v], nat[y]]],
          w -> nat[z]}]

Out[8]= or[member[nat[z], ld[nat[x], nat[y]]],
        not[equal[natadd[nat[z], natmul[nat[v], nat[y]]],
                   natmul[nat[u], nat[x]]]]] = True

In[9]:= or[member[nat[z_], ld[nat[x_], nat[y_]]],
        not[
          equal[natadd[nat[z_], natmul[nat[v_], nat[y_]]],
                  natmul[nat[u_], nat[x_]]]]] = True
\end{verbatim}