asymmetric part of a transitive relation

Johan G. F. Belinfante
2009 May 19

\begin{verbatim}
In[1]:= SetDirectory["l:" ]; << goedel.09may18a; << tools.m

:Package Title: goedel.09may18a
2009 May 18 at 5:55 a.m.

It is now:  2009 May 19 at 13:29

Loading Simplification Rules

TOOLS.M Revised 2009 May 16

weightlimit = 40
\end{verbatim}

\textbf{summary}

The asymmetric part of a transitive relation is transitive:

\begin{verbatim}
In[2]:= implies[TRANSITIVE[x], TRANSITIVE[dif[x, inverse[x]]]]
\end{verbatim}


In this notebook a variable-free expression of this fact is derived for the special case of transitive relations that are sets. A particularly nice formulation of this uses the idempotent function \texttt{composite[DIF, id[INVERSE], inverse[FIRST]]}.

\textbf{derivation}

The best results are obtained if the function \texttt{IMAGE[SWAP]} which takes \texttt{x} to \texttt{inverse[x]} is replaced with its restriction \texttt{INVERSE}.

Lemma. The class of transitive relations is contained in the class of relations whose asymmetric part is transitive.

\begin{verbatim}
In[3]:= Map[subclass[TRV, intersection[P[cart[V, V]], #]] &,
              union[complement[TRV], fix[composite[
                  inverse[image[inverse[DIF], TRV]], IMAGE[SWAP]]]] // Normality] // InvertFix
\end{verbatim}

Out[3]= subclass[TRV, fix[composite[INVERSE, image[inverse[DIF], TRV]]]] = True

\begin{verbatim}
In[4]:= % /. Equal -> SetDelayed
\end{verbatim}

Lemma. The asymmetric part of a transitive relation is transitive.
\textbf{Lemma.} Simplification rule.

\texttt{In[7]:=} \texttt{fix[complement[\text{inverse}[E]], \text{INVERSE}, \text{id}[\text{TRV}], E]} // \text{Normality}\\
\texttt{Out[7]=} \texttt{fix[complement[\text{inverse}[E]], \text{INVERSE}, \text{id}[\text{TRV}], E]} \texttt{=} \texttt{Di}\\

\textbf{Lemma.} An irreflexive transitive relation is its own asymmetric part.

\texttt{In[9]:=} \texttt{SubstTest[implies, subclass[y, fix[\text{funpart}[x]]],}\\
\quad \quad \texttt{equal[\text{image}[\text{funpart}[x], y], y],}\\
\quad \quad \quad \texttt{\{x \rightarrow \text{composite}[\text{DIF}, \text{id}[\text{INVERSE}], \text{inverse}[\text{FIRST}]],}\\
\quad \quad \quad \quad \quad \texttt{y \rightarrow \text{TRV}, z \rightarrow \text{TRV}\}}\\
\texttt{Out[9]=} \texttt{equal[\text{image}[\text{DIF, composite}[\text{INVERSE, id}[\text{TRV}]]],}\\
\quad \quad \texttt{\{z \rightarrow \text{TRV}\}}\\
\texttt{Lemma.} Every irreflexive transitive relation is the asymmetric part of a transitive relation.

\texttt{In[11]:=} \texttt{SubstTest[implies, subclass[u, v], subclass[\text{image}[t, u], \text{image}[t, v]],}\\
\quad \quad \texttt{\{t \rightarrow \text{composite}[\text{DIF, id}[\text{INVERSE}], \text{inverse}[\text{FIRST}]],}\\
\quad \quad \quad \texttt{u \rightarrow \text{intersection}[\text{TRV, P[Di]}],}\\
\quad \quad \quad \quad \quad \texttt{v \rightarrow \text{TRV}\}}\\
\texttt{Out[11]=} \texttt{subclass[\text{intersection}[\text{TRV, P[Di]}], \text{image}[\text{DIF, composite}[\text{INVERSE, id}[\text{TRV}]]]]}\\
\texttt{\texttt{Lemma.} The class of asymmetric parts of transitive relations is equal to the class of irreflexive transitive relations.}