constant unary and binary operations

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In[1]: = SetDirectory["l:"]; << goedel82.10b; << tools.m

:Package Title: goedel82.10b 2006 June 10 at 10:20 p.m.

It is now: 2006 Jun 11 at 18:22

Loading Simplification Rules

TOOLS.M Revised 2006 June 6

weightlimit = 40

summary

Formulas for non-empty constant unary and binary operations are derived.

lemmas

In[2]: = AssInt[range[CART], range[CART], FUNS]

Out[2]= intersection[CONST, range[CART]] = CONST

In[3]: = intersection[CONST, range[CART]] := CONST

In[4]: = AssInt[range[CART], FUNS, UNOPS]


In[5]: = intersection[UNOPS, range[CART]] := intersection[CONST, UNOPS]

In[6]: = AssInt[range[CART], FUNS, BINOPS]


In[7]: = intersection[BINOPS, range[CART]] := intersection[BINOPS, CONST]

constant unary operations

Theorem.
Reln Normality:

\[
\text{ImageComp}[\text{CART}, \text{inverse}[\text{CART}], \text{UNOPS}] = \text{cart}[V, \text{set}[0]], \text{cart}[\text{set}[0], V], \text{composite}[\text{inverse}[\text{E}], \text{IMAGE}[\text{SINGLETON}]]
\]

A formula for nonempty constant unary operations can be obtained from this.

\[
\text{Map}[\text{equal}[\text{image}[\text{CART}, \text{composite}[\text{inverse}[\text{E}], \text{IMAGE}[\text{SINGLETON}]]], \text{dif}[\#, \text{set}[0]]] \&, \text{ImageComp}[\text{CART}, \text{inverse}[\text{CART}], \text{UNOPS}]]
\]

Corollary.

\[
\text{SubstTest}[U, \text{image}[\text{CART}, x], x \rightarrow \text{composite}[\text{inverse}[\text{E}], \text{IMAGE}[\text{SINGLETON}]]]
\]

Theorem.

\[
\text{image}[\text{inverse}[\text{CART}], \text{BINOPS}] // \text{RelnNormality}
\]

A formula for nonempty constant binary operations is obtained the same way as for unary operations.

\[
\text{Map}[\text{equal}[
\text{image}[\text{CART}, \text{composite}[\text{inverse}[\text{E}], \text{IMAGE}[\text{SINGLETON}], \text{inverse}[\text{DUP}], \text{inverse}[\text{CART}]]],
\text{dif}[\#, \text{set}[0]]] \&, \text{ImageComp}[\text{CART}, \text{inverse}[\text{CART}], \text{BINOPS}]]
\]

\[
\text{image}[\text{inverse}[\text{CART}], \text{BINOPS}] := \text{union}[\text{cart}[V, \text{set}[0]], \text{cart}[\text{set}[0], V],
\text{composite}[\text{inverse}[\text{E}], \text{IMAGE}[\text{SINGLETON}], \text{inverse}[\text{DUP}], \text{inverse}[\text{CART}]]]
\]
Corollary.

\[
\text{In [18]} := \text{SubstTest}[U, \text{image}[\text{CART}, x],
\quad x \rightarrow \text{composite}[\text{inverse}[E], \text{IMAGE}[\text{SINGLETON}], \text{inverse}[\text{DUP}], \text{inverse}[\text{CART}]]]
\]

\[
\text{Out [18]} = U[\text{intersection}[\text{BINOPS}, \text{CONST}]] = \text{cart}[\text{cart}[V, V], V]
\]

\[
\text{In [19]} := U[\text{intersection}[\text{BINOPS}, \text{CONST}]] := \text{cart}[\text{cart}[V, V], V]
\]