epsilon induction

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In this notebook, the proof of theorem TC-REG-2 obtained by Otter on 2001 March 24 is rederived. This theorem is similar to what Thomas Jech calls proof by epsilon induction. See Theorem 25(A) on page 73 of the following reference:

The notation and terminology differs slightly. What is here called the full class \( x \) he calls a transitive class, which he denotes by \( T \). Jech talks about a property \( \Phi \) instead of the class \( y \) of sets that satisfy some property. Jech assumes that the axiom of regularity holds, in which case \( \text{REGULAR} = V \), so one can omit the intersection with the class \( \text{REGULAR} \) that appears in theorem TC-REG-2. Jech adds an unnecessary hypothesis that the empty set belongs to the class \( y \). When the axiom of regularity holds, any non-empty full class \( x \) holds the empty set. If \( x \) is not empty, the condition \( \text{subclass}[\text{intersection}[x,P[y]],y] \) implies that the empty set belongs to \( y \). When \( x \) is empty, the theorem holds trivially.

main argument

Lemma.
Main part of Otter's argument.

Elimination of the variable \( z \).

the rest of the story

The following special case is available, and will be used to prove the general case.

A straightforward substitution yields a result with the desired conclusion, but a different hypothesis.
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