HULL[x] and INVERSE

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(summary)
A theorem about composites of HULL[x] with INVERSE is derived.

(derivation)
The following characterization of HULL functions will be used.

\( \text{implies[and[FUNCTION[x], idempotent[x], subclass[x, S], subcommute[x, S]], equal[x, HULL[fix[x]]]]} \)

The function to which this theorem will be applied is composite[INVERSE, HULL[x], INVERSE].

\( \text{SubstTest[implies, subclass[u, v], subclass[image[w, u], image[w, v]],} \)
\( \{u \rightarrow \text{HULL[x]}, v \rightarrow S, w \rightarrow \text{cross[INVERSE, INVERSE]}} \)\)

The following new rule helps in this regard.

\( \text{composite[S, INVERSE]} \)
A corollary is that `INVERSE` subcommutes with `S`.

A second application yields:

At this point, one obtains:

The following theorem will be used to obtain a simple corollary.
Corollary. The functions \( \text{HULL}[Z] \) and \( \text{INVERSE} \) commute.

\[
\text{SubstTest}[\text{implies}, \text{equal}[u, v], \text{equal}[\text{composite}[w, u], \text{composite}[w, v]],
\{u \to \text{composite}[\text{INVERSE}, \text{HULL}[Z], \text{INVERSE}], v \to \text{HULL}[Z], w \to \text{INVERSE}\}]
\]

\[
\text{equal}[\text{composite}[\text{INVERSE}, \text{HULL}[Z]], \text{composite}[\text{HULL}[Z], \text{INVERSE}]] = \text{True}
\]
The following orientation of this equation as a rewrite rule is tentative, based on an analogy with a similar rule for HULL[TRV].

\[ \text{In[28]} := \text{composite[HULL[Z], INVERSE]} := \text{composite[INVERSE, HULL[Z]]} \]