sethood of image[RANK, x]

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2006 March 2

In[1]: = SetDirectory["l:"]; << goedel79.01a; << tools.m

:Package Title: goedel79.01a

2006 March 1 at 6:30 p.m.

It is now: 2006 Mar 2 at 21:11

Loading Simplification Rules

TOOLS.M Revised 2006 February 3

weightlimit = 40

summary

Although the RANK function is not one-to-one, its inverse is thin:

In[2]: = thin[inverse[RANK]]


This fact is exploited in this notebook to derive a sethood rule for image[RANK, x]. The class of the ranks of the regular members of a class x is a set if and only if the class of the regular members of x is a set.

derivation

Since the domain of RANK is the class REGULAR, one has:

In[3]: = SubstTest[implies, subclass[u, v], subclass[image[u, x], image[v, x]],
               {u -> id[REGULAR], v -> composite[inverse[RANK], RANK]}]

Out[3]= subclass[intersection[REGULAR, x], image[inverse[RANK], image[RANK, x]]] = True

In[4]: = (\% /. \ x \_ \_ ) /. Equal \rightarrow SetDelayed

Using the subset theorem yields:

In[5]: = SubstTest[implies, and[subclass[u, v], member[v, V]], member[u, V],
               {u -> intersection[REGULAR, x], v -> image[inverse[RANK], image[RANK, x]]}]

Out[5]= or[member[intersection[REGULAR, x], V],
        not[member[image[inverse[RANK], image[RANK, x]], V]]] = True
Combining the above results yields an implication in one direction:

\[ \text{In [7]} := \text{Map[not, SubstTest[and, implies[p1, p2, implies[p2, p3], not[implies[p1, p3]],}
\]
\[ \begin{align*}
&\{p1 \rightarrow \text{member[image[RANK, x], V]},
&p2 \rightarrow \text{member[image[inverse[RANK], image[RANK, x]], V]},
&p3 \rightarrow \text{member[intersection[REGULAR, x], V]]]}\}
\]
\[\text{Out[7]} = \text{or[member[intersection[REGULAR, x], V], not[member[image[RANK, x], V]]]} = \text{True} \]

\[ \text{In [8]} := \text{(% /. \_ x \_) / . Equal \rightarrow \text{SetDelayed}} \]

Since imaging with the \text{RANK} function simply ignores the irregular members, an application of the axiom of replacement yields a reverse implication:

\[ \text{In [9]} := \text{SubstTest[implies, member[y, V], member[image[RANK, y], V], y \rightarrow \text{intersection[REGULAR, x]]}} \]
\[\text{Out[9]} = \text{or[member[image[RANK, x], V], not[member[intersection[REGULAR, x], V]]]} = \text{True} \]

\[ \text{In [10]} := \text{(% /. \_ x \_) / . Equal \rightarrow \text{SetDelayed}} \]

The sethood rule is obtained by combining the above implications:

\[ \text{In [11]} := \text{equiv[member[image[RANK, x], V], member[intersection[REGULAR, x], V]]} \]
\[\text{Out[11]} = \text{True} \]
\[\text{In [12]} := \text{member[image[RANK, x], V]} := \text{member[intersection[REGULAR, x], V]} \]