the integer one divides all integers

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In[1]:= SetDirectory["1:“]; << goedel88.18a; << tools.m

:Package Title: goedel88.18a 2006 December 18 at 9:00 a.m.
It is now: 2006 Dec 18 at 9:34
Loading Simplification Rules
TOOLS.M Revised 2006 December 17
weightlimit = 40

summary

In this notebook it is shown that every integer is divisible by the integer +1 (= plus[set[0]] = composite[id[omega], SUCC]). To make sense of what is going on here, one must understand that the integer divisibility relation INTDIV has been defined as the union of all endomorphisms of INTADD. At this point, the theory of multiplication for integers has not yet been developed, and indeed the result derived here is needed for that development.

derivation

By definition, the integer divisibility relation is:

In[2]:= U[binhom[INTADD, INTADD]]

Out[2]= INTDIV

Since the sum of two endomorphisms is another, the integer divisibility relation satisfies a corresponding property:

In[3]:= ImageComp[cross[inverse[DUP], INTADD], inverse[E], image[CROSS, cart[binhom[INTADD, INTADD], binhom[INTADD, INTADD]]]] // Reverse
Out[3]= composite[INTADD, intersection[composite[inverse[FIRST], INTDIV], composite[inverse[SECOND], INTDIV]]] = INTDIV

In[4]:= composite[INTADD, intersection[composite[inverse[FIRST], INTDIV], composite[inverse[SECOND], INTDIV]]] := INTDIV

Corollary. The set image[INTDIV,set[x]] of integers divisible by x is closed under addition.
Lemma.

\[ \text{In} [7] := \text{SubstTest[member, pair[x, x], composite[Id, w], w \rightarrow \text{INTDIV}]} \]

\[ \text{Out}[7] := \text{and[member[x, V], member[pair[x, x], \text{INTDIV}]] := member[pair[x, x], \text{INTDIV}]} \]

Theorem. Every integer is indivisible by itself.

\[ \text{In} [9] := (\text{member[x, fix[y]] // AssertTest // Reverse} /. y \rightarrow \text{INTDIV}) \]

\[ \text{Out}[9] := \text{member[pair[x, x], \text{INTDIV}] := member[x, Z]} \]

A form of induction suitable for integers is applied to the class of integers divisible by one.

\[ \text{In} [11] := \text{SubstTest[implies, and[member[composite[id[omega], SUCC], x], subclass[image[INTADD, cart[x, x]], x], subclass[image[INVERSE, x], x]], subclass[Z, x], x \rightarrow image[\text{INTDIV}, set[plus[set[0]]]]]} \]

\[ \text{Out}[11] := \text{subclass[Z, image[\text{INTDIV}, set[composite[id[omega], SUCC]]]]} = \text{True} \]

Theorem. Every integer is indivisible by +1.

\[ \text{In} [13] := \text{SubstTest[and, subclass[u, v], subclass[v, u], [u \rightarrow Z, v \rightarrow image[\text{INTDIV}, set[composite[id[omega], SUCC]]]]]} \]

\[ \text{Out}[13] := \text{equal[Z, image[\text{INTDIV}, set[composite[id[omega], SUCC]]]]} = \text{True} \]

\[ \text{In} [14] := \text{image[\text{INTDIV}, set[composite[id[omega], SUCC]]]} := Z \]