quasigroup isotopes

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In[1]:= << l:goedel.09feb04a; << l:tools.m

:Package Title: goedel.09feb04a 2009 February 4 at 4:10 p.m.
It is now:  2009 Feb 5 at 7:28
Loading Simplification Rules
TOOLS.M  Revised 2009 February 4
weightlimit = 40

summary

In general an isotope of a quasigroup $q$ is a function of the form $p_1 \circ q \circ (p_2 \otimes p_3)$, where $p_1$, $p_2$ and $p_3$ are permutations of $\text{range}[q]$. Since the rotation of a quasigroup is a quasigroup, the general case can be reduced to the special case that $p_2$ and $p_3$ are $\text{id}[\text{range}[q]]$.

sethood rewrite rule

In this section a convenient sethood rule is derived for composites.

Theorem.  A conditional rewrite rule.

In[2]:= \text{implies} [\text{and} [\text{member} [x, V], \text{member} [y, V], \text{member} [\text{composite} [x, y], V]]]


In[3]:= \text{member} [\text{composite} [\_\_, y\_\_], V] := \text{True} /\!\!\!\text{; member} [x, V] \&\& \text{member} [y, V]

Example:

In[4]:= \text{member} [\text{rotate} [\text{composite} [\text{perm} [x], \text{quasigp} [y]]], V]


derivation

Theorem.
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Lemma.

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variable-free reformulation

Theorem. The variables can be eliminated using `RelnNormality` as follows, yielding an inclusion. In the remainder of this section, this inclusion will be sharpened to an equation.

```
In[21]:= Map[empty[composite[Id, complement[#1]]] &,
      dif[composite[id[QUASIGPS], inverse[IMAGE[SECOND]]], IMAGE[FIRST], id[PERMS]],
      image[inverse[COMPOSE], QUASIGPS]] // complement // RelnNormality
Out[21]= subclass[image[COMPOSE, composite[id[QUASIGPS],
      inverse[IMAGE[SECOND]]], IMAGE[FIRST], id[PERMS]]], QUASIGPS] = True
```

Lemma.
Lemma. The \texttt{quasigp} wrapper and the variable \texttt{x} in the lemma can be eliminated at the same time using \texttt{reify}, yielding an inclusion in the opposite direction.

In[24] := \% ./ \texttt{x} \rightarrow \texttt{x}_. / . \texttt{Equal} \rightarrow \texttt{SetDelayed}

Theorem. The two inclusions are combined into an equation which is made into a rewrite rule.

In[26] := \% ./ \texttt{Equal} \rightarrow \texttt{SetDelayed}