monotone relations are functions

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In[1]:= SetDirectory["1:"]; << goedel.10aug06a;<< tools.m

:Package Title: goedel.10aug06a 2010 August 6 at 8:00 p.m.

It is now: 2010 Aug 7 at 8:38

Loading Simplification Rules

TOOLS.M Revised 2010 July 26

weightlimit = 40

summary

Monotone relations are functions. This interesting fact was discovered in the course of trying to understand a recently derived theorem about monotone functions. A generalization of the following recently derived theorem is obtained, dropping the hypothesis that \( x \) be a function.

In[85]:= implies[and[FUNCTION[x], subclass[composite[x, S, inverse[x]], S]],
  subclass[x, composite[S, CORE[fix[x]]]]]

Out[85]= True

It was then discovered that the appearance of having generalized the existing theorem is in fact illusory because the monotonicity hypothesis implies that \( \text{Id} \circ x \) is a function.

derivation

Lemma.

In[76]:= SubstTest[implies, subclass[u, v], subclass[composite[u, w], composite[v, w]],
  \{u \rightarrow \text{composite}[x, S, \text{id}[y]], v \rightarrow S, w \rightarrow E\} // Reverse

Out[76]= or[not[subclass[composite[x, S, id[y]], S]],
  subclass[composite[x, inverse[\text{CORE}[y]]], S]] := True

In[77]:= or[not[subclass[composite[x_, S, id[y_]], S]],
  subclass[composite[x_, inverse[\text{CORE}[y_]]], S]] := True

Lemma.
The appearance of having generalized the existing theorem is illusory because the monotonicity requirement implies that \(\text{Id} \circ x\) is a function. This fact will now be derived.

Theorem.

\[\text{Map}[\text{not}, \text{SubstTest}[\text{and}, \text{implies}[\text{p1}, \text{p2}], \text{implies}[\text{p2}, \text{p3}]], \text{not}[\text{implies}[\text{p1}, \text{p3}]], [\text{p1} \rightarrow \text{subclass}[\text{composite}[x, S, \text{inverse}[x]], S]], \text{p2} \rightarrow \text{subclass}[\text{composite}[x, S, \text{id}[\text{fix}[x]]], S], \text{p3} \rightarrow \text{subclass}[\text{composite}[\text{Id}, x], \text{composite}[S, \text{CORE}[\text{fix}[x]]]])] \text{ // Reverse}
\]

\[\text{subclass}[\text{composite}[\text{Id}, x], \text{composite}[S, \text{CORE}[\text{fix}[x]]]]] = \text{True}
\]

\[\text{subclass}[\text{composite}[\text{Id}, x], \text{composite}[S, \text{CORE}[\text{fix}[x]]]]] = \text{True}
\]

The appearance of having generalized the existing theorem is illusory because the monotonicity requirement implies that \(\text{Id} \circ x\) is a function. This fact will now be derived.

Theorem.

\[\text{SubstTest}[\text{subclass}, t, \text{intersection}[u, v]], \text{t} \rightarrow \text{composite}[x, \text{inverse}[x]], \text{u} \rightarrow S, v \rightarrow \text{inverse}[S])
\]

\[\text{subclass}[\text{composite}[x, \text{inverse}[x]], S] = \text{FUNCTION}[\text{composite}[\text{Id}, x]]
\]

\[\text{subclass}[\text{composite}[x, \text{inverse}[x]], S] = \text{FUNCTION}[\text{composite}[\text{Id}, x]]
\]

Theorem. A variable-free restatement of the theorem that monotone relations are functions.

\[\text{Map}[\text{equal}[V, \#] \&, \text{dif}[\text{cliques}[\text{complement}[\text{cross}[S, \text{complement}[S]]]]], \text{image}[\text{inverse}[\text{IMAGE}[\text{id}[\text{cart}[V, V]]]], \text{FUNS}]] \text{ // complement} \text// \text{Normality}
\]

\[\text{subclass}[\text{intersection}[\text{cliques}[\text{complement}[\text{cross}[S, \text{complement}[S]]]]], \text{P}[\text{cart}[V, V]]], \text{FUNS} = \text{True}
\]
In[105]:= 
    subclass[intersection[
        cliques[complement[cross[S, complement[S]]]], P[cart[V, V]], FUNS] := True