summary

The composite of \texttt{CORE[RFX]} and \texttt{CORE[SYM]} in either order is equal to \texttt{CORE[intersection[RFX,SYM]]}.

commute

The reflexive core of the symmetric core is equal to the symmetric core of the reflexive core.

\begin{verbatim}
In[2]:= core[RFX, core[SYM, x]] == core[SYM, core[RFX, x]]

\end{verbatim}

From this one can deduce that the functions \texttt{CORE[RFX]} and \texttt{CORE[SYM]} commute.

\begin{verbatim}
In[3]:= symdif[composite[core[RFX], core[SYM]],
    composite[core[SYM], core[RFX]]] // VSNormality

    composite[core[SYM], core[RFX]]],
    intersection[composite[complement[core[SYM]], core[RFX]],
    composite[core[RFX], core[SYM]]]] == 0
\end{verbatim}

\begin{verbatim}
In[4]:= . Equal -> SetDelayed
\end{verbatim}
Corollary. The composite is idempotent.

The following temporary lemmas will all be subsumed by later rules.

Lemma.

The following will be needed later.
uniqueness theorem for CORE

The uniqueness theorem for CORE now implies:

\[
\text{In [16]:=} \quad \text{SubstTest[implies, and[FUNCTION[x], idempotent[x], subcommute[x, S], subclass[x, inverse[S]], equal[V, domain[x]]], equal[x, CORE[fix[x]]], x \to \text{composite[CORE[RFX, CORE[SYM]]}]}
\]

\[
\text{Out[16]=} \quad \text{equal[composite[CORE[RFX, CORE[SYM]], CORE[intersection[RFX, SYM]]] = True}
\]

\[
\text{In [17]:=} \quad \text{composite[CORE[RFX, CORE[SYM]] := CORE[intersection[RFX, SYM]]}
\]

\[
\text{In [18]:=} \quad \text{composite[CORE[SYM], CORE[RFX]] // VSNormality}
\]

\[
\text{Out[18]=} \quad \text{composite[CORE[SYM], CORE[RFX]] = CORE[intersection[RFX, SYM]]}
\]

\[
\text{In [19]:=} \quad \text{composite[CORE[SYM], CORE[RFX]] := CORE[intersection[RFX, SYM]]}
\]

image rules

The following general result holds:

\[
\text{In [20]:=} \quad \text{ImageComp[CORE[x], CORE[x], V] // Reverse}
\]

\[
\text{Out[20]=} \quad \text{image[CORE[x], Uclosure[x]] = Uclosure[x]}
\]

\[
\text{In [21]:=} \quad \text{image[CORE[x_], Uclosure[x_]] := Uclosure[x]}
\]

Corollary. (Comment: The corresponding result for SYM is already in the GOEDEL program.)

\[
\text{In [22]:=} \quad \text{SubstTest[image, CORE[x], Uclosure[x], x \to RFX]}
\]

\[
\text{Out[22]=} \quad \text{image[CORE[RFX], RFX] = RFX}
\]

\[
\text{In [23]:=} \quad \text{image[CORE[RFX], RFX] := RFX}
\]

Other such rules:

\[
\text{In [24]:=} \quad \text{ImageComp[CORE[RFX], CORE[SYM], V] // Reverse}
\]

\[
\text{Out[24]=} \quad \text{image[CORE[RFX], SYM] = intersection[RFX, SYM]}
\]
In[25]:= image[CORE[RFX], SYM] := intersection[RFX, SYM]

In[26]:= ImageComp[CORE[SYM], CORE[RFX], V] // Reverse

Out[26]= image[CORE[SYM], RFX] == intersection[RFX, SYM]

In[27]:= image[CORE[SYM], RFX] := intersection[RFX, SYM]