T2 and transvar

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2014 February 12

In[1]:= SetDirectory["l:" ]; << goedel.14feb09a

:Package Title: goedel.14feb09a 2014 February 9 at 5:50 p.m.

Loading takes about seventeen minutes, half that time due to builtin pauses.

It is now: 2014 Feb 12 at 12:6

Loading Simplification Rules

TOOLS.M is now incorporated in the GOEDEL program as of 2010 September 3

weightlimit = 40

Loading completed.

It is now: 2014 Feb 12 at 12:24

summary

The T2 separation condition in topology is technically complicated because this condition involves four occurrences of the variable for the topology.

In[2]:= equiv[member[t, T2], and[member[t, V], subclass[composite[id[U[t]], Di, id[U[t]]],
composite[inverse[E], id[t], DISJOINT, id[t], E]]]] // not // not


One way to simplify this condition is to introduce the cartesian square of t as a new variable x = t \times t. The condition on t can then be rewritten as a condition on x that involves only two occurrences of the variable x. This observation is used to derive a formula for T2 involving a transvar[x, y] expression. A consequence is that the class T2 is closed under unions of chains.

derivation

Lemma.
Theorem. A formula for T2.

SubstTest`subclass, Uchains`, fix[image`inverse`[CART, t]],
fix[image`inverse`[CART, Uchains`, t]], t → Uclosure`x` // Reverse

out[5]=

Lemma.

SubstTest`subclass, Uchains`, fix[image`inverse`[CART, t]],
fix[image`inverse`[CART, Uchains`, t]], t → Uclosure`x` // Reverse

out[10]=

Theorem.

equal[Uchains`, fix[image`inverse`[CART, Uclosure`x`]]],
fix[image`inverse`[CART, Uclosure`x`]] // AssertTest

out[14]=

Corollary.

SubstTest`Uchains`,
fix[image`inverse`[CART, Uclosure`t`]], t → transvar`x, y` // Reverse

out[18]=

out[19]=
Theorem. The class $T_2$ is closed under chain-unions.

In[21]:= SubstTest[Uchains, fix[image[inverse[CART], transvar[x, y]]],
{x -> composite[inverse[E], IMAGE[id[D]], CART],
y -> composite[inverse[E], CART, id[DISJOINT]]}] // Reverse


In[22]:= Uchains[T2] := T2

**comment**

Observation. The following holds, but no interesting consequences of it were obtained.

In[30]:= intersection[image[CART, Id], transvar[composite[inverse[E], IMAGE[id[D]], CART],
composite[inverse[E], CART, id[DISJOINT]]]]

Out[30]= image[CART, id[T2]]

Observation. One can use this to deduce the following fact, but this result is already known.

In[31]:= SubstTest[member, cart[x, x], intersection[u, v],
{u -> image[CART, Id], v -> transvar[composite[inverse[E], IMAGE[id[D]], CART],
composite[inverse[E], CART, id[DISJOINT]]]]

Out[31]= and[member[x, V], subclass[composite[id[U[x]], Di, id[U[x]]],
composite[inverse[E], id[x], DISJOINT, id[x], E]] = member[x, T2]

Theorem. An amusing result.

In[34]:= SubstTest[implies, and[equal[Uchains[x], x], equal[Uchains[y], y]],
equal[Uchains[intersection[x, y], intersection[x, y]]],
{x -> image[CART, Id], y -> transvar[composite[inverse[E], IMAGE[id[D]], CART],
composite[inverse[E], CART, id[DISJOINT]]]]} // Reverse

Out[34]= equal[image[CART, id[T2]], Uchains[image[CART, id[T2]]]] = True

In[36]:= Uchains[image[CART, id[T2]]] := image[CART, id[T2]]